Why or Why not Wye-Wye?

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Alexandria VA

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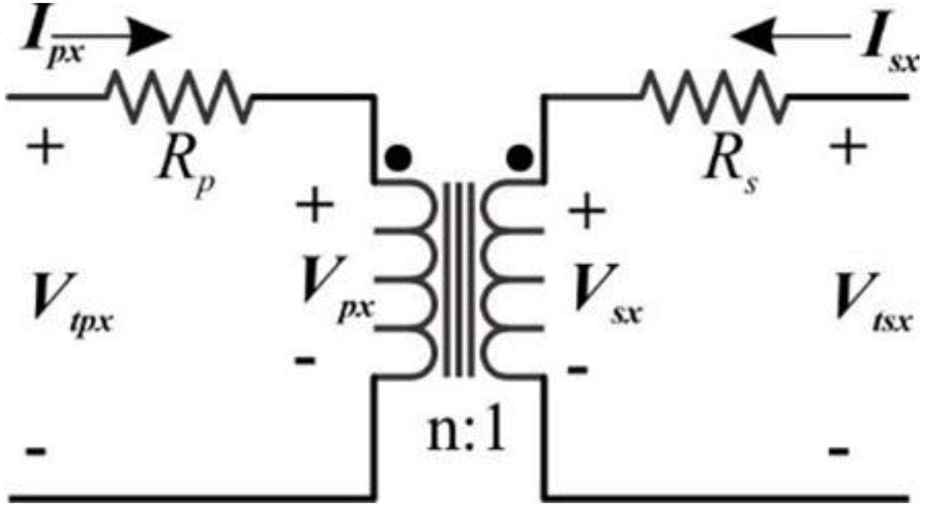
Agenda

- Single phase transformers
- Three phase power review
- Common-Mode fundamentals
- Delta-Delta transformers
- Delta-Wye transformers
- Wye-Delta transformers
- Wye-Wye transformers

Single Phase transformers

- Three phase transformers usually constructed from three single phase transformers



$$V_{px} = nV_{sx}$$

$$nI_{px} = -I_{sx}$$

$$V_{tpx} = V_{px} + R_p I_{px}$$

$$V_{tsx} = V_{sx} + R_s I_{sx}$$

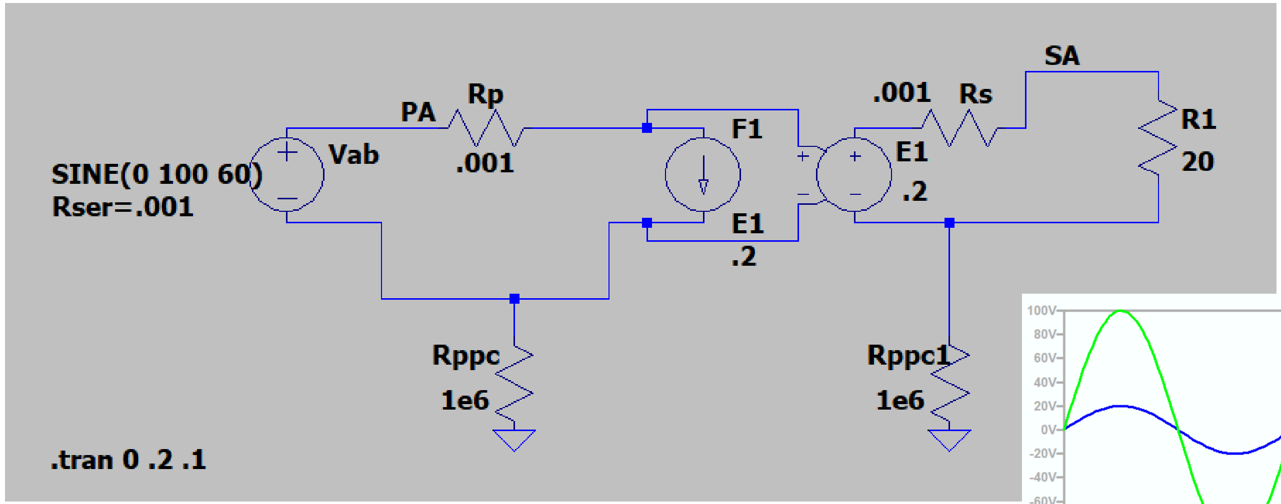
$$R_T = R_p + R_s\, n^2$$

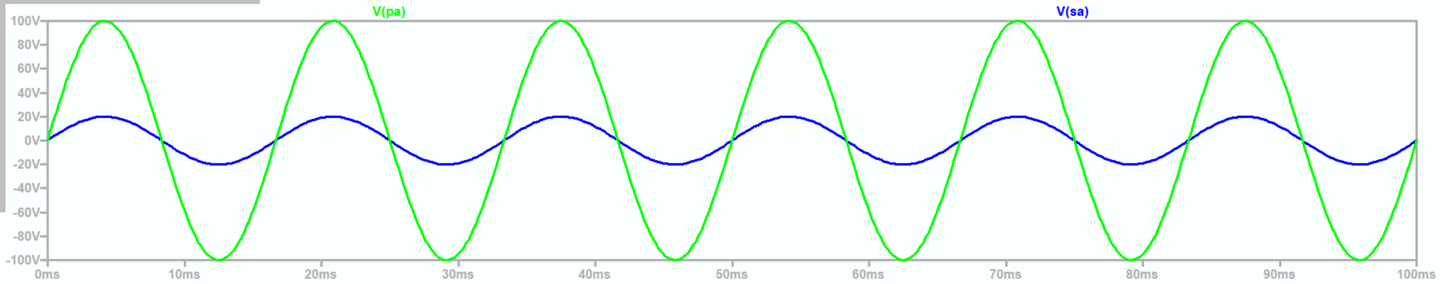
$$V_{tsx} = \frac{1}{n} V_{tpx} + \frac{1}{n^2} R_T I_{sx}$$

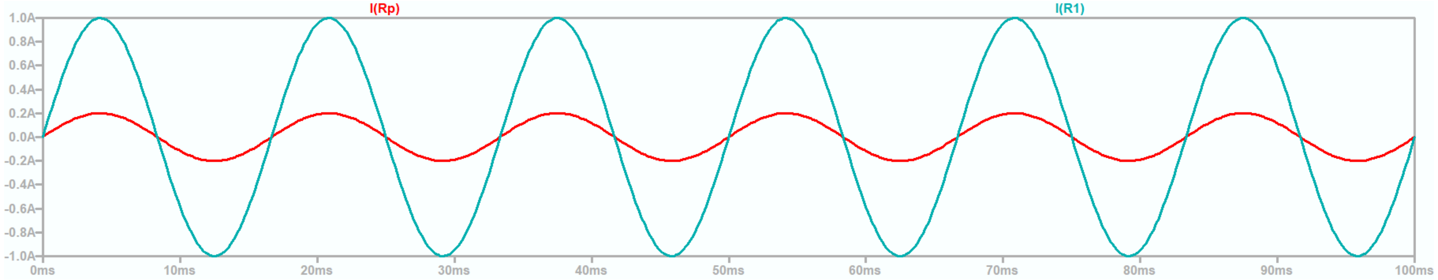
$$I_{sx} = -nI_{px}$$

Transformer Regulation Term

Modeling single phase transformer in LTSpice







Voltage Controlled Voltage Source (Gain = 1/n)
Current Controlled Current Source (Gain = 1/n)

Note: Every separately derived system requires a connection to ground (Rppc and Rppc1)

Single Phase Transformer modeling nuances

- Magnetization current is in addition to ideal transformer primary current.
  - One may often ignore this current if the questions of concern are entirely on the secondary side.
- Turns ratio is more closely equal to the ratio of the voltages than to the ratio of the currents.
- In addition to the series resistance, there can also be a series inductance.
- Nonlinearities can result in current (and voltage) harmonics.
- Parasitic capacitance exists between windings and ground; and between windings.

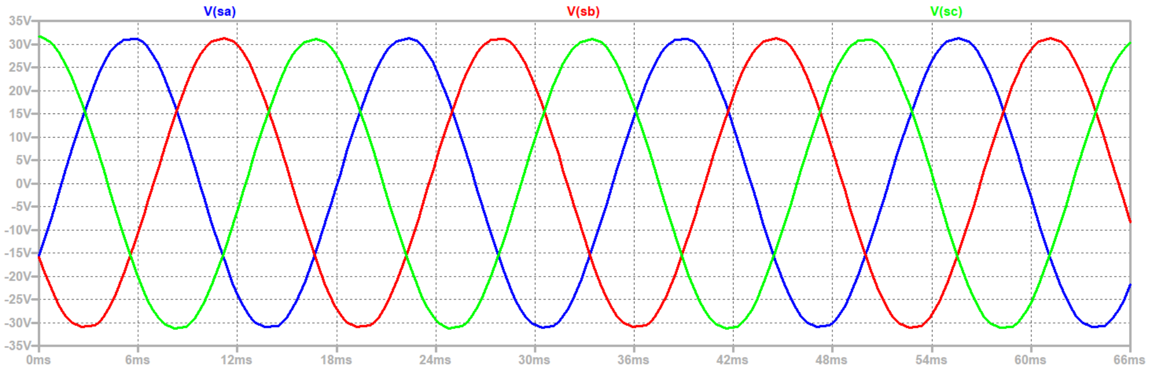
Three Phase Power Review

- Balanced three-phase = waveforms with equal magnitude and frequency but phase shifted by 120 electrical degrees

$$V_{an} = V_{peak}\sin(\omega t)$$

$$V_{bn} = V_{peak}\sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$V_{cn} = V_{peak}\sin\left(\omega t + \frac{2\pi}{3}\right)$$



- The three waveforms sum to zero

$$V_{an} + V_{bn} + V_{cn} = V_{peak}\sin(\omega t) + V_{peak}\sin\left(\omega t - \frac{2\pi}{3}\right) + V_{peak}\sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$= V_{peak}\left(\sin(\omega t) + \sin(\omega t)\cos\left(-\frac{2\pi}{3}\right) + \cos(\omega t)\sin\left(-\frac{2\pi}{3}\right) + \sin(\omega t)\cos\left(\frac{2\pi}{3}\right) + \cos(\omega t)\sin\left(\frac{2\pi}{3}\right)\right)$$

$$= V_{peak}\left(\sin(\omega t) - \sin(\omega t)\frac{1}{2} - \sin(\omega t)\frac{1}{2}\right) = 0$$

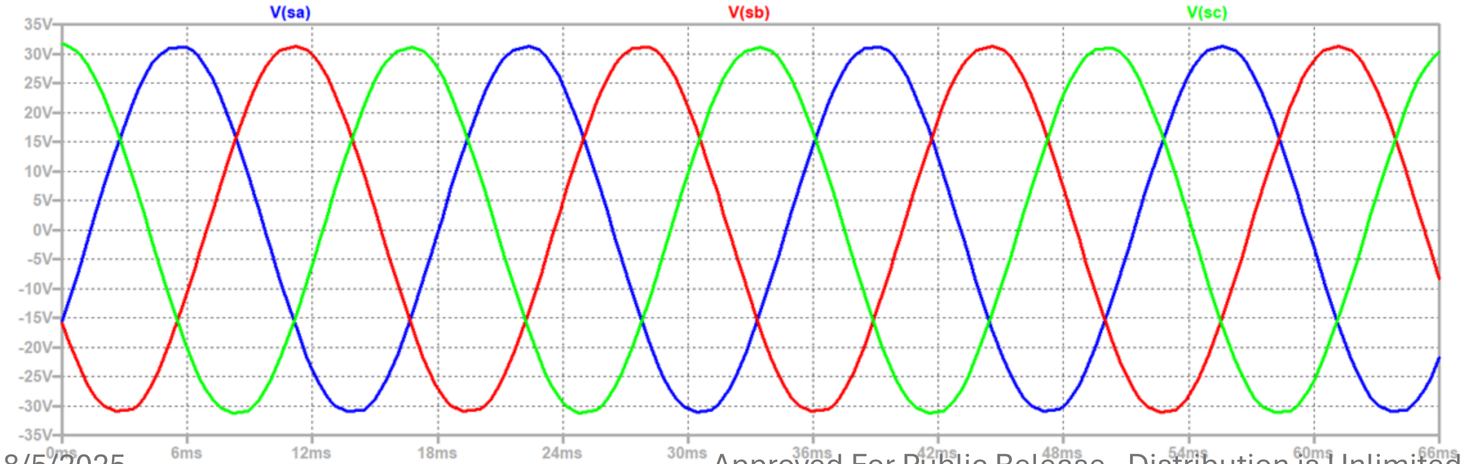
Three Phase Power Review

Peak value is $\sqrt{2}$ times the root mean square (RMS) value

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t=0}^{t=T} \left(V_{peak}\sin(\omega t)\right)^2 dt}$$

$$V_{rms} = \sqrt{\frac{V_{peak}^2}{T\omega}\left|\frac{\omega t}{2} - \frac{1}{2}(\sin(\omega t)\cos(\omega t))\right|_0^T} = \frac{V_{peak}}{\sqrt{2}}$$

$$V_{peak} = \sqrt{2}V_{rms}$$



$$V_{lnPeak} \approx 32\ volts$$

$$V_{lnRMS} \approx \frac{32}{\sqrt{2}} \approx 22.6\ volts$$

Three Phase Power Review

$V_{ll} \approx 55\ volts\ peak$
$V_{ln} \approx 32\ volts\ peak$

$$\frac{55}{32} \approx 1.72$$

$$\sqrt{3} \approx 1.73$$

- Line to line voltage is $\sqrt{3}$ times the line to neutral voltage

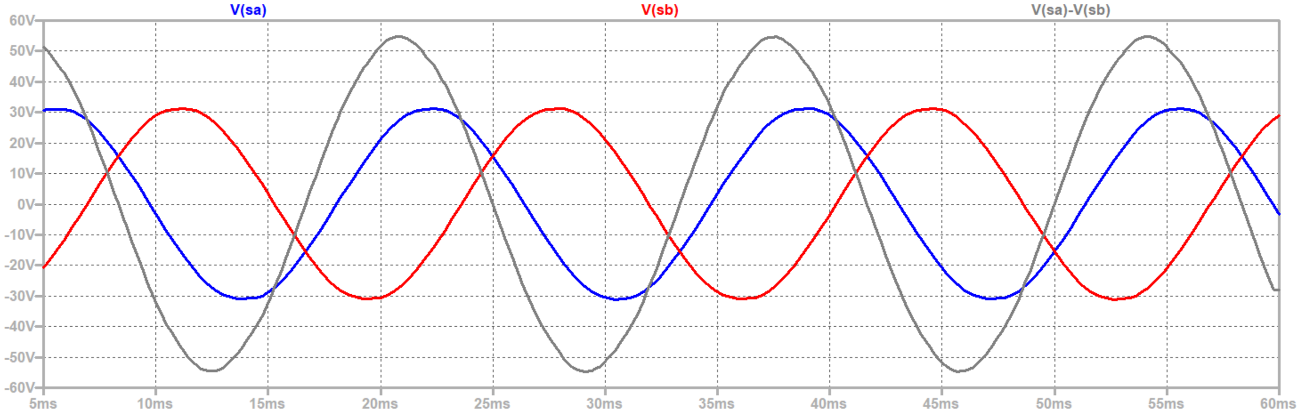
$$V_{an} = V_{peak}\sin(\omega t) \quad V_{bn} = V_{peak}\sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$V_{ab} = V_{an} - V_{bn} = V_{peak}\left(\sin(\omega t) - \sin\left(\omega t + \frac{2\pi}{3}\right)\right) = V_{peak}\left(\sin(\omega t) - \sin(\omega t)\cos\left(\frac{2\pi}{3}\right) - \cos(\omega t)\sin\left(\frac{2\pi}{3}\right)\right)$$

$$V_{ab} = V_{peak}\left(\frac{3}{2}\sin(\omega t) - \cos(\omega t)\frac{\sqrt{3}}{2}\right)$$

$$|V_{ab}| = V_{peak}\sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = V_{peak}\sqrt{3}$$

- Phase shift is $\frac{\pi}{6}$ radians (from V_{an}) or 30 degrees



Three Phase Power Review

Real Power

$$P = V_{an}I_a + V_{bn}I_b + V_{cn}I_c$$

$$P = V_{mag}\sin(\omega t)\,I_{mag}\sin(\omega t + \theta) + V_{mag}\sin\left(\omega t - \frac{2\pi}{3}\right)I_{mag}\sin\left(\omega t - \frac{2\pi}{3} + \theta\right) + V_{mag}\sin\left(\omega t + \frac{2\pi}{3}\right)I_{mag}\sin\left(\omega t + \frac{2\pi}{3} + \theta\right)$$

After a lot of trigonometry ….

$$P = \frac{3}{2}V_{mag}I_{mag}\cos(\theta)$$

Convert from peak to rms values for both current and voltage

$$P = 3V_{\mathrm{lnrms}}I_{rms}\cos(\theta)$$

Convert from line to neutral voltage to line to line voltage

$$P = \sqrt{3}V_{\mathrm{llrms}}I_{rms}\cos(\theta)$$

$\cos(\theta)$ = Power Factor

$$V_{llrms} \approx 39\ volts$$

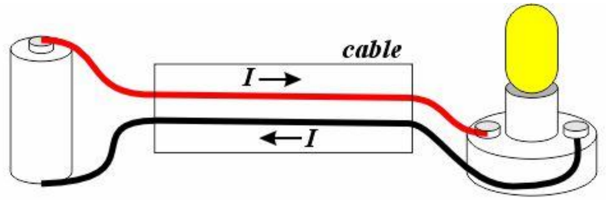
$$I_{rms} \approx 0.23\ amps$$

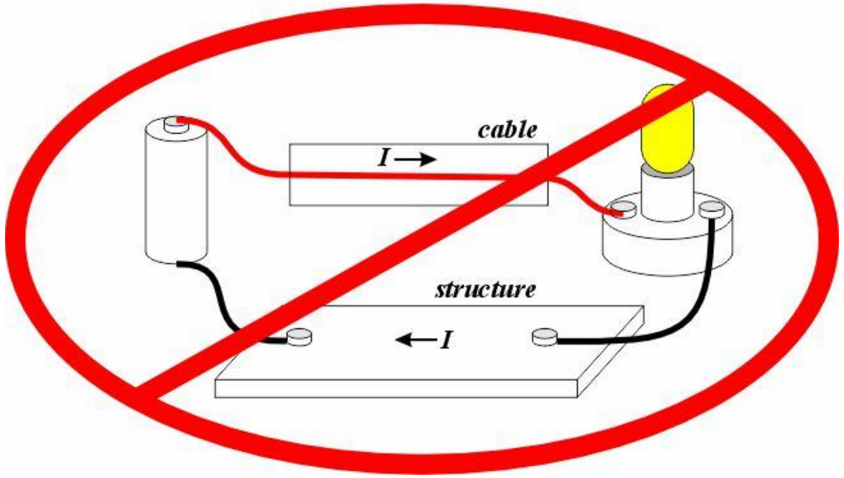
$$P \approx 15.5\ Watts$$

Common-Mode fundamentals

Shipboard power systems are designed for the "Differential Mode"

- Power is distributed via a set of conductors
  - The current going to a load or another power system component is intended to return via the same set of conductors.
    - Unlike automobiles which use structure as a current return path.
  - Common Mode Current is the sum of the current flowing in the same direction through the set of conductors
    - CM Current is usually intended to equal zero.
    - CM Currents in real systems may not be zero.



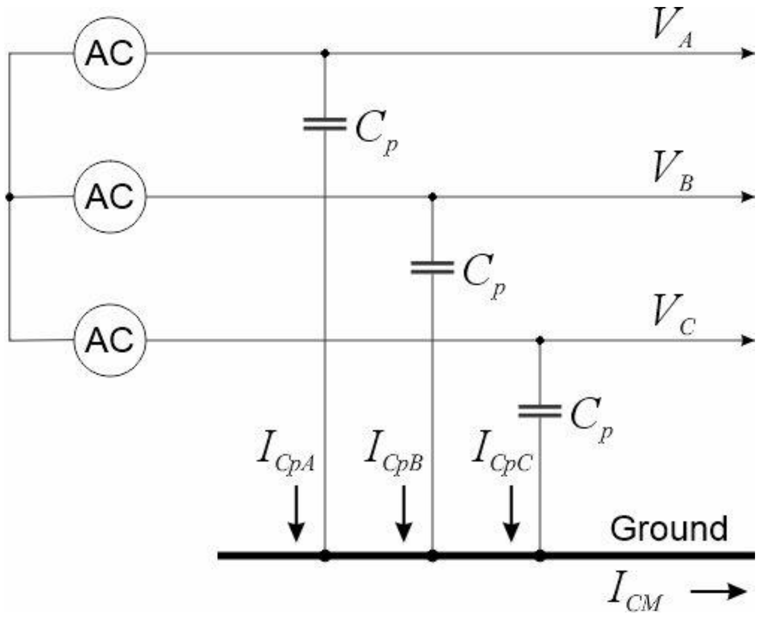


Common Mode Current Properties

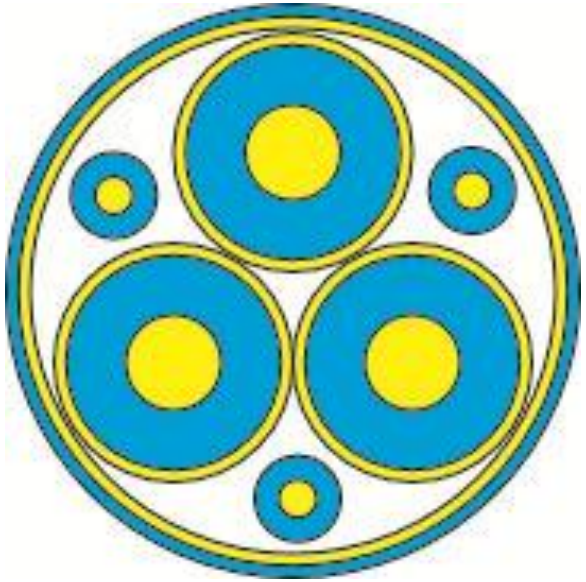
- Common mode currents are usually orders of magnitude less than the desired differential mode currents.
- Loads connected only between phases and without connections to ground do not directly contribute to common mode currents (Kirchhoff's current law prevents this).
  - Can be delta or wye (with wye-point unterminated)
- Sources connected only between phases and without connections to ground do not directly contribute to common mode currents (Kirchhoff's current law prevents this).
  - Can be delta or wye (with wye-point unterminated)
- Series impedances (resistance, inductance, capacitance) impact common mode currents.

Common Mode Voltage properties

- Common mode voltage (neutral voltage) is a property of a set of conductors.
  - Measured with respect to a specified reference potential (such as ground or another neutral).
  - Each conductor voltage is measured instantaneously, then averaged across conductors.
  - Generally, cannot measure it directly in a circuit, must calculate it from measurements on each of the conductors – or insert a network to derive the neutral.
  - A conductor labeled as a Neutral may or may not be at the neutral voltage (common mode voltage) .
- No common mode current can flow from the set of parasitic capacitances if a common mode voltage with respect to ground is not present on the conductors.



Common Mode current paths

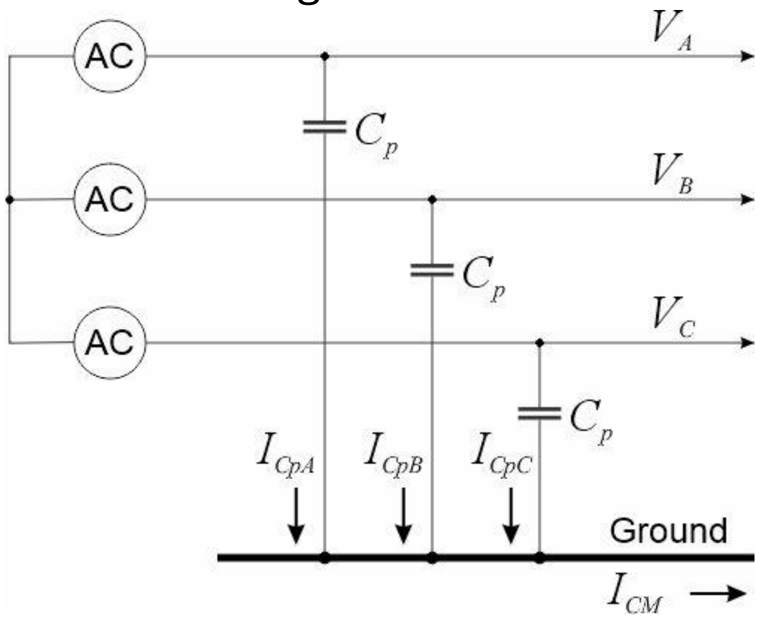


- AC power components typically have parasitic capacitances.
  - Cable conductors and their grounded shields / drain wires
  - Cable conductors and ship structure
  - Transformer / generator / motor winding to winding capacitance
  - Transformer / generator / motor winding to ground capacitance
- Grounding system may also provide a path for currents through the ship structure.
  - Solidly Grounded
  - High Resistance Grounded Systems
- System configurations can provide multiple current paths.
  - Ring bus
  - Multiple auctioneering diode connected loads in dc systems

Three Phase Parasitic Capacitance

- The Neutral Voltage (V_N) of a set conductors is the average value of the conductor voltages with respect to a reference voltage (such as ground).

The Neutral Voltage is also called a Common Mode Voltage



$$V_N = \frac{1}{3}(V_A + V_B + V_C)$$

$$V_A = V_N + V_{AN}$$

$$V_B = V_N + V_{BN}$$

$$V_C = V_N + V_{CN}$$

$$V_N = \frac{1}{3}(V_N + V_{AN} + V_N + V_{BN} + V_N + V_{CN})$$

$$V_{AN} + V_{BN} + V_{CN} = 0$$

$$I_{CpA} = C_p\left(\frac{dV_N}{dt} + \frac{dV_{AN}}{dt}\right)$$

$$I_{CpB} = C_p\left(\frac{dV_N}{dt} + \frac{dV_{BN}}{dt}\right)$$

$$I_{CpC} = C_p\left(\frac{dV_N}{dt} + \frac{dV_{CN}}{dt}\right)$$

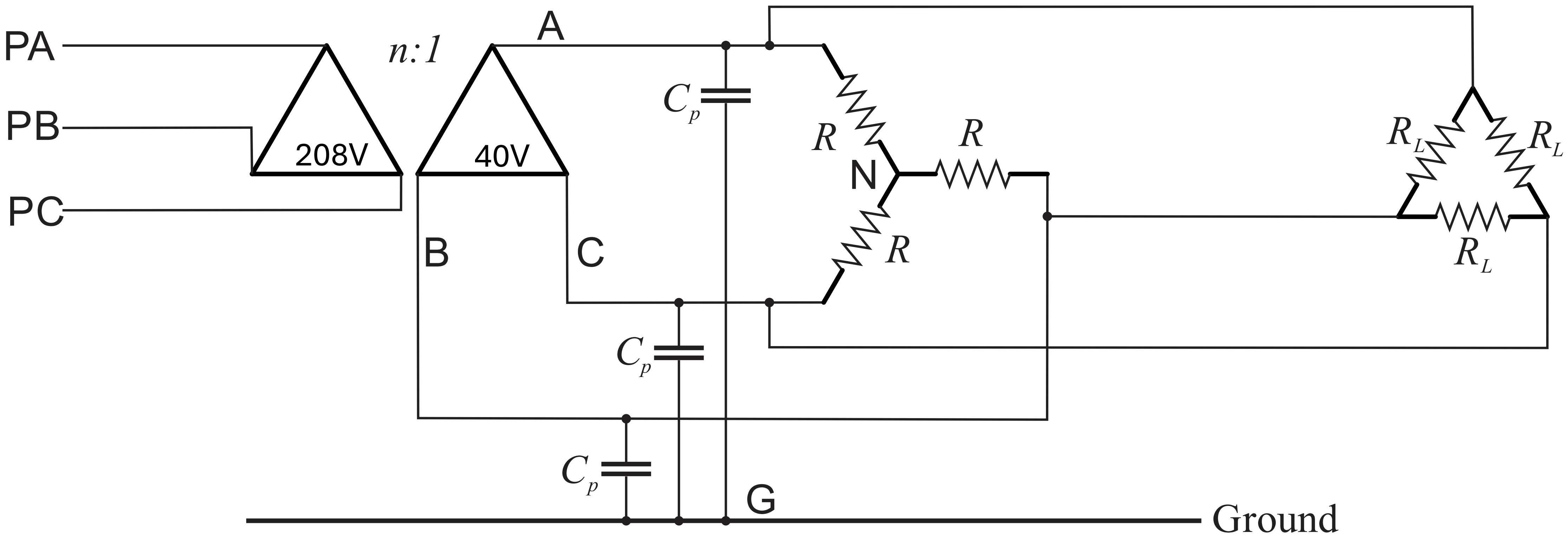
$$I_{CM} = I_{CpA} + I_{CpB} + I_{CpC}$$

$$I_{CM} = 3C_p\left(\frac{dV_N}{dt}\right) + C_p\left(\frac{d(V_{AN} + V_{BN} + V_{CN})}{dt}\right)$$

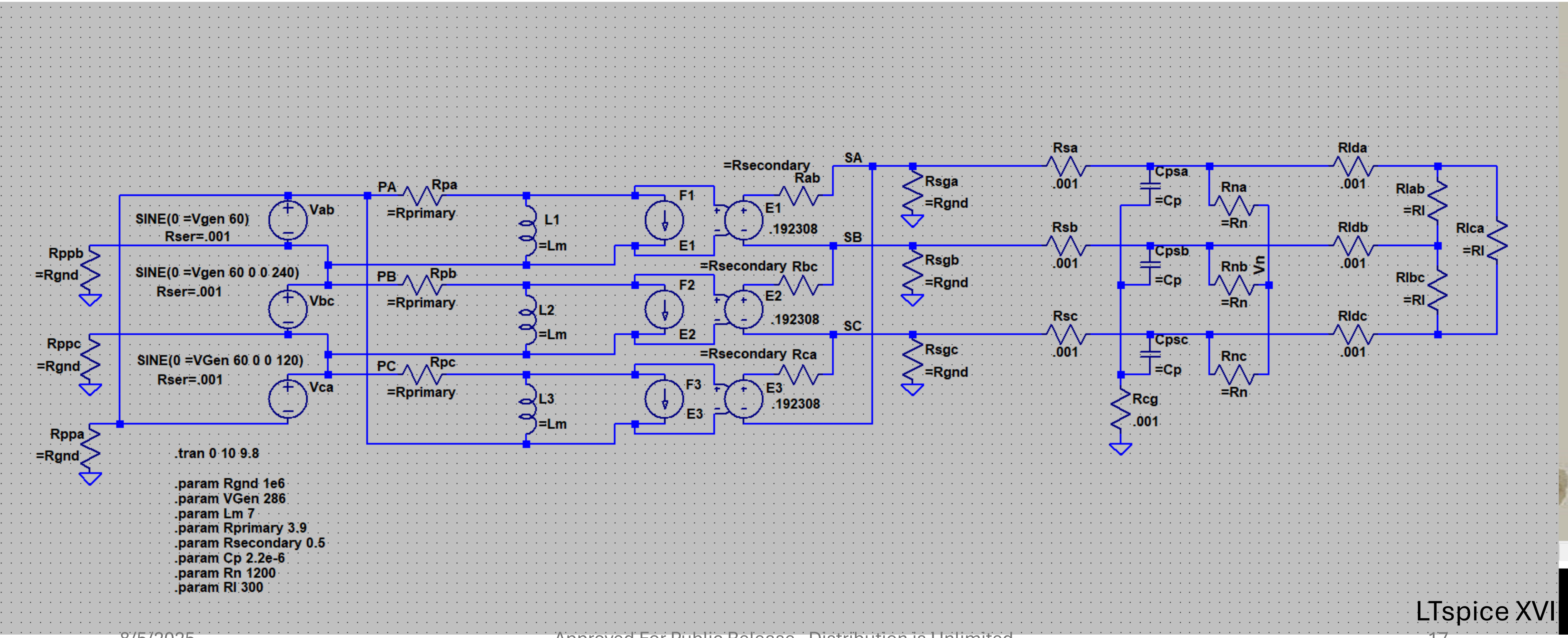
$$I_{CM} = 3C_p\left(\frac{dV_N}{dt}\right)$$

Common Mode Current only Depends on the Neutral Voltage and parasitic capacitance

Delta-Delta Transformers Ungrounded secondary



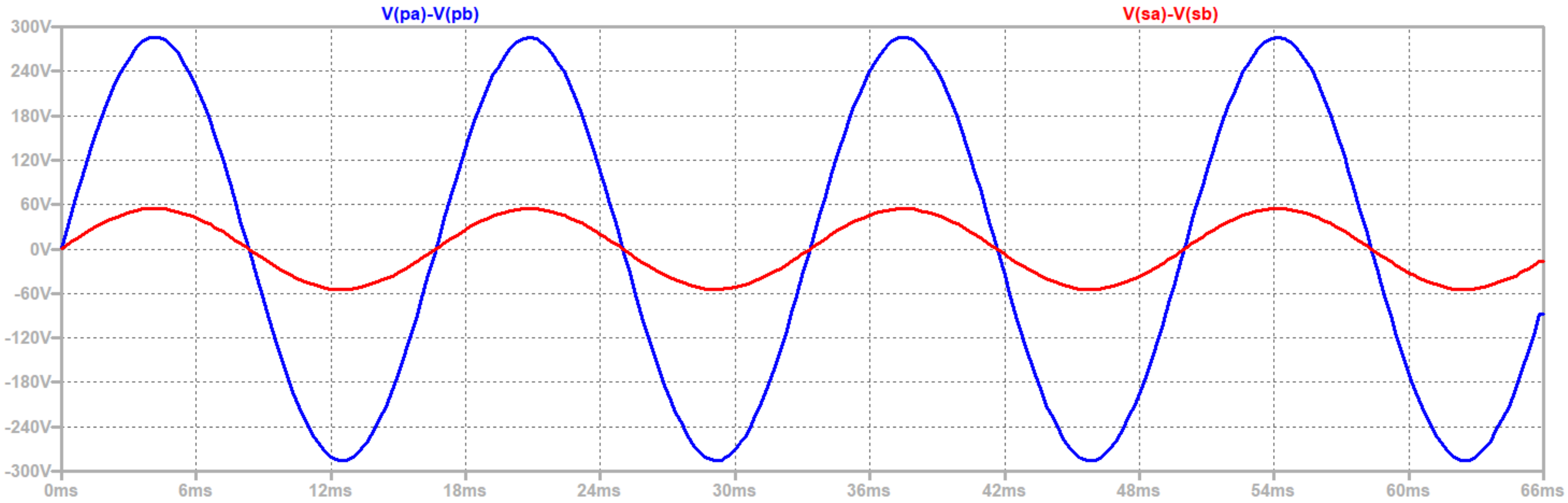
Simulation Model – Delta-Delta



LTspice XVI

20240727 experiment 1-nl.asc

Transformer turns ratio is approximately the ratio of primary to secondary voltage (Delta-Delta)



$$n_{rating} = \frac{208}{40} = 5.2$$

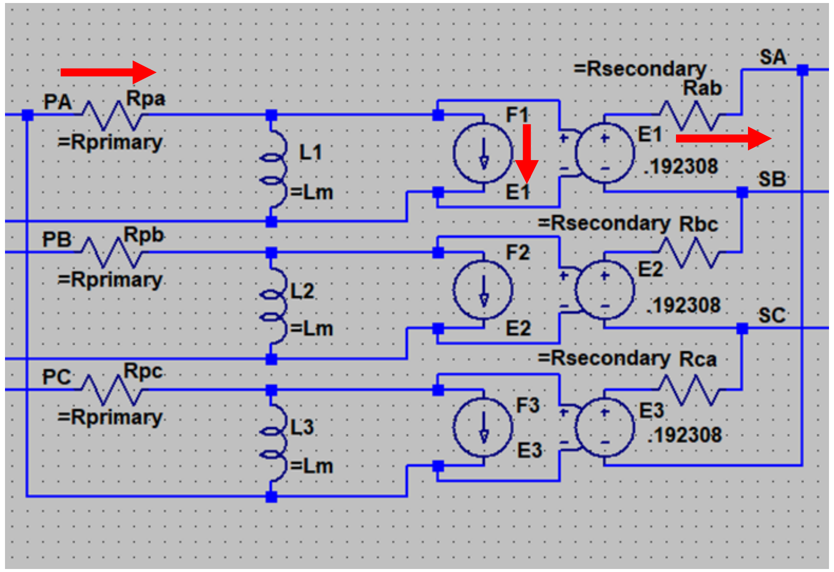
$$n_{simulated} = \frac{287}{55} \approx 5.2$$

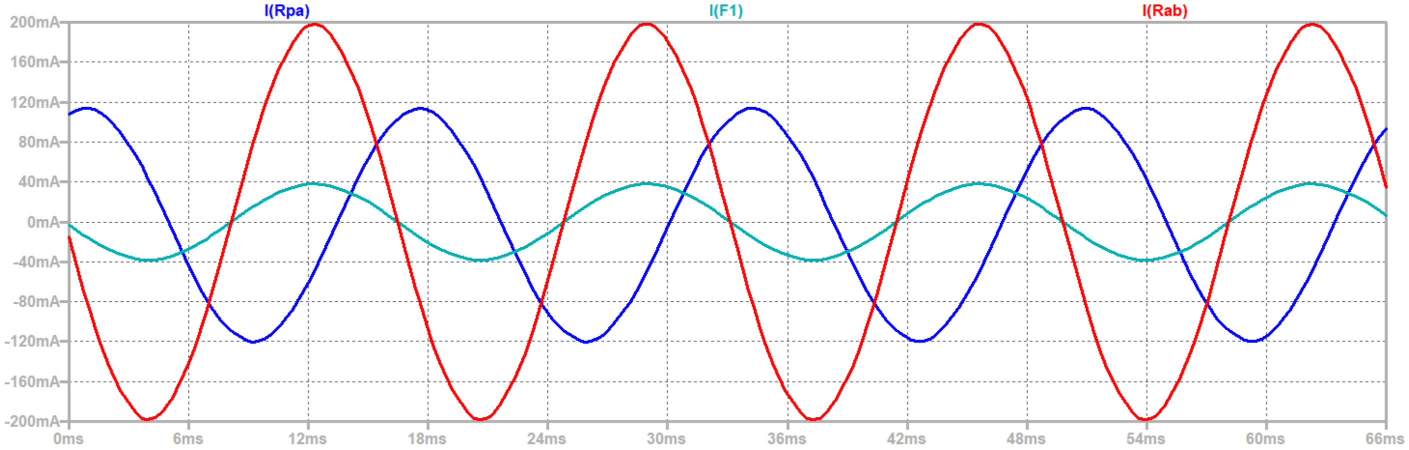
Ratio of currents not exactly the turns ratio (Delta – Delta)

$$n_{rating} = \frac{208}{40} = 5.2$$

$$n_{Ideal\_simulated} = \frac{200}{39} \approx 5.1$$

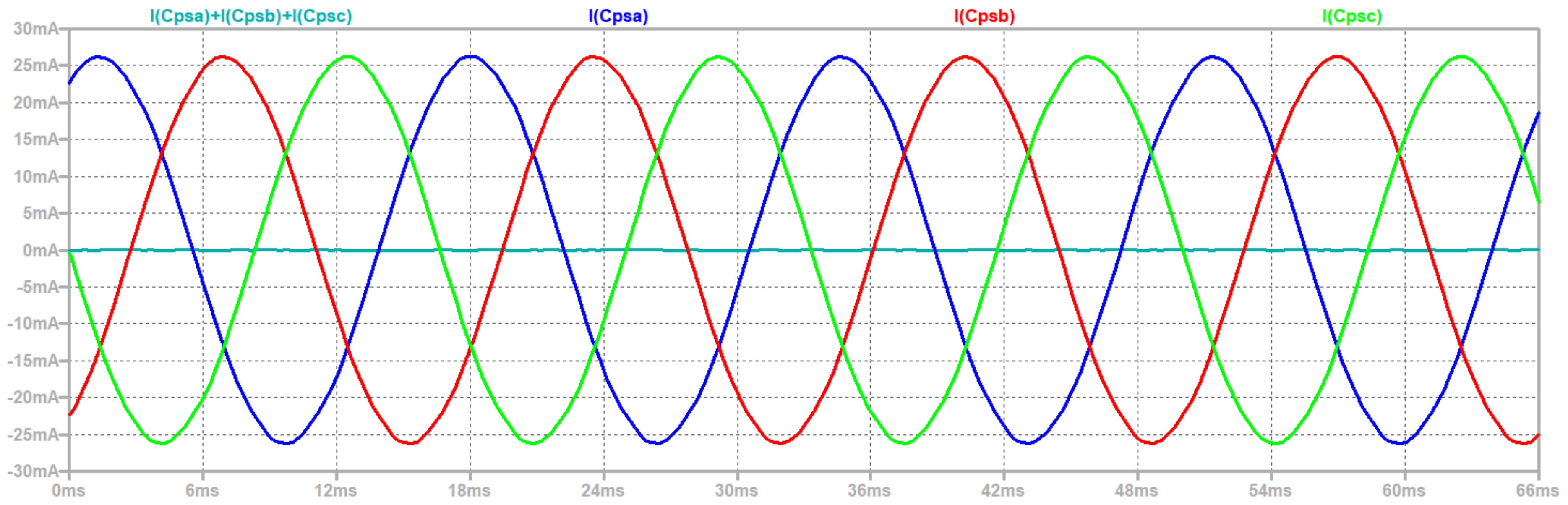
$$n_{simulated} = \frac{200}{114} \approx 1.8$$





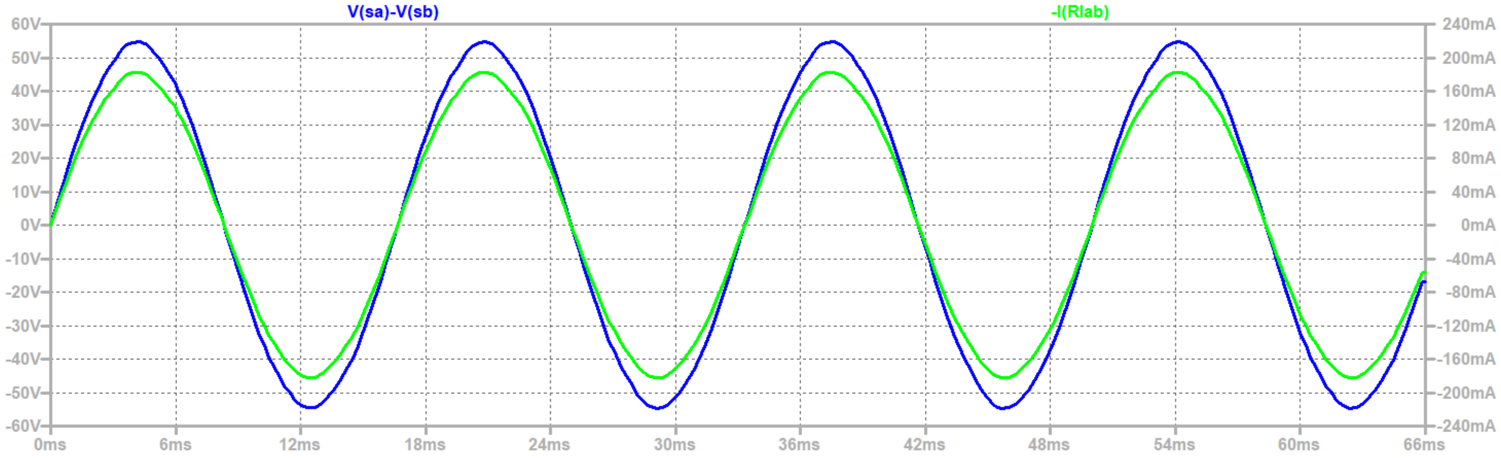
Approved For Public Release. Distribution is Unlimited

The sum of the currents through the parasitic capacitances is zero (in balanced operation – Delta-Delta)



$I_{peak} \approx 26\ mA$

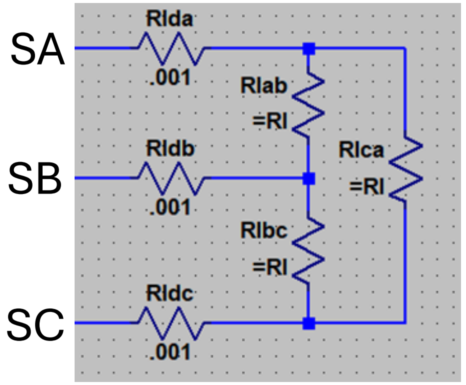
Current in Load Resistor



$$I_{RLa} = \frac{V_{SA} - V_{SB}}{R_L}$$

$$I_{RLa\_simulated} = \frac{55}{300} = .183\ amps\ peak$$

$$I_{RLa\_simulated} = \frac{55}{300\sqrt{2}} = .130\ amps\ rms$$



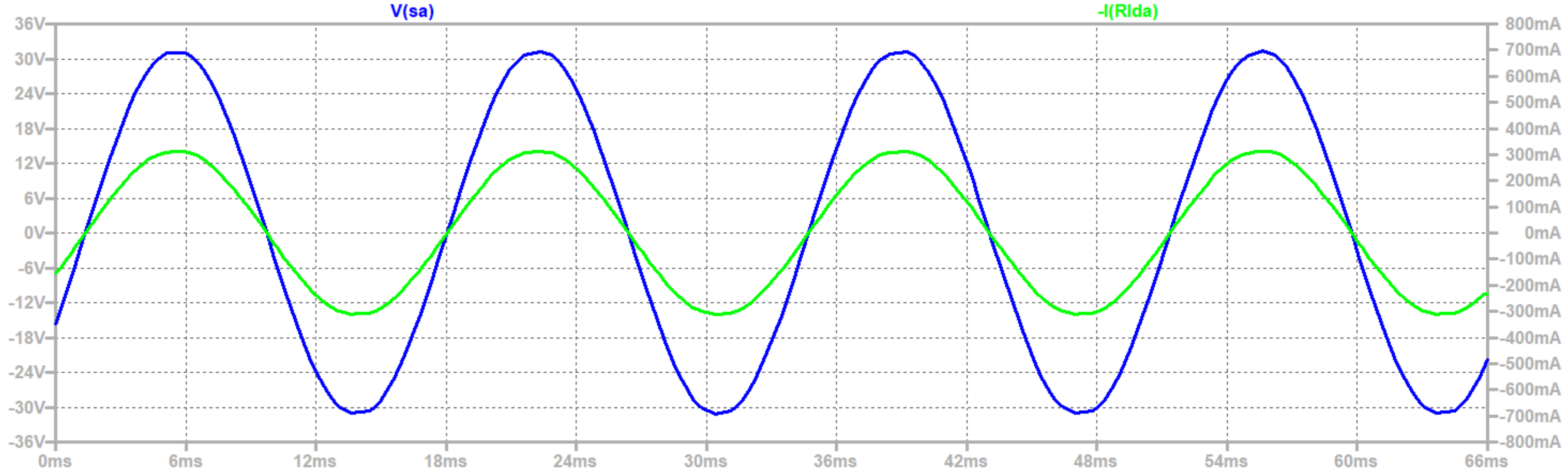
Line current into Delta resistive Load

$$I_a = \frac{V_{SA} - V_{SB}}{R_L} + \frac{V_{SA} - V_{SC}}{R_L}$$

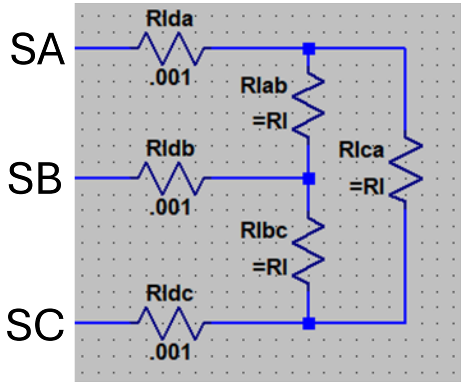
$$I_a = \frac{V_{Ln}}{R_L}\left(\sin(\omega t) - \sin\left(\omega t - \frac{2\pi}{3}\right) + \sin(\omega t) - \sin\left(\omega t + \frac{2\pi}{3}\right)\right)$$

$$I_a = \frac{V_{Ln}}{R_L}\left(2\sin(\omega t) - \sin(\omega t)\cos\left(-\frac{2\pi}{3}\right) - \cos(\omega t)\sin\left(-\frac{2\pi}{3}\right) - \sin(\omega t)\cos\left(\frac{2\pi}{3}\right) - \cos(\omega t)\sin\left(\frac{2\pi}{3}\right)\right)$$

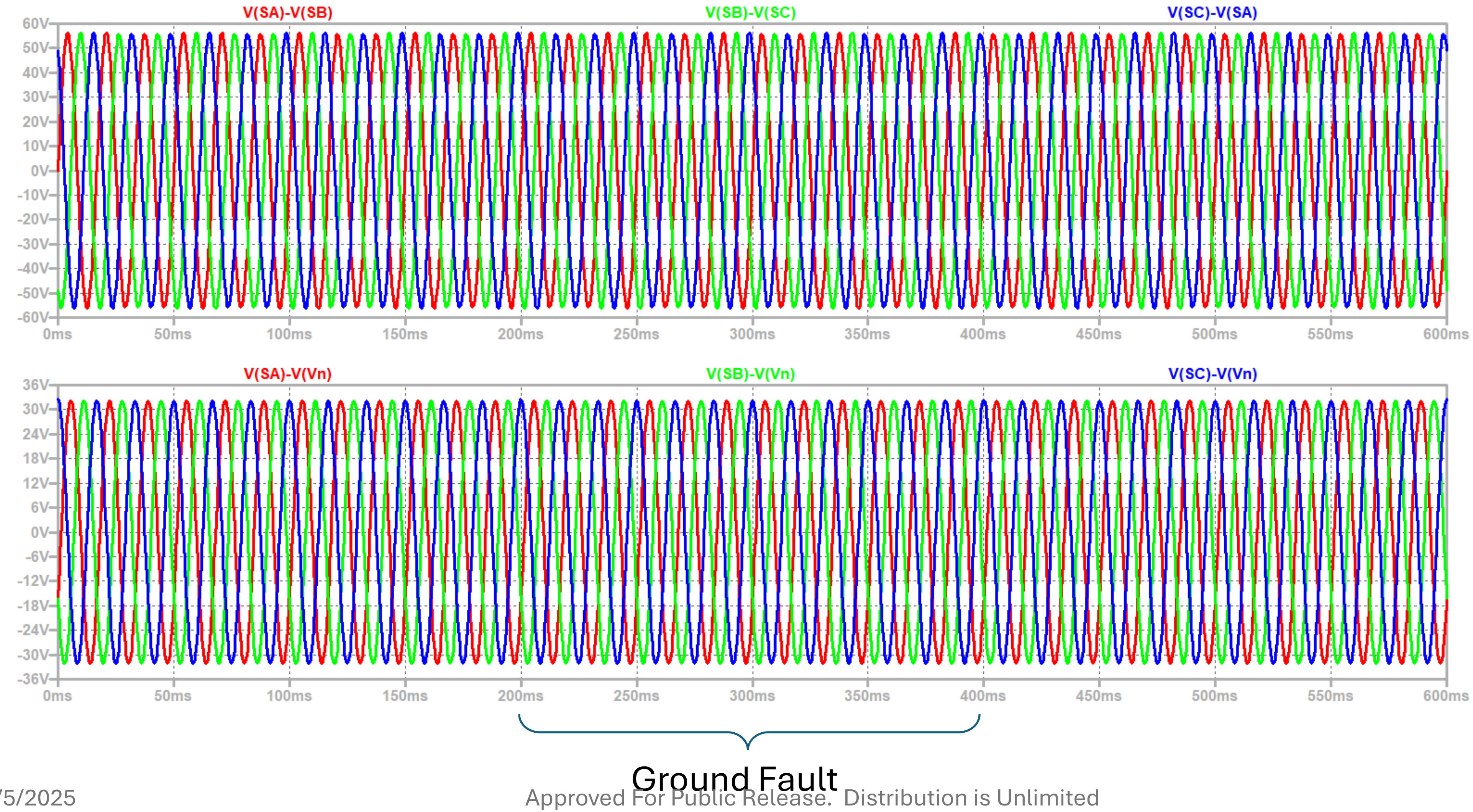
$$I_a = \frac{V_{Ln}}{R_L}\left(2\sin(\omega t) + \frac{1}{2}\sin(\omega t) + \frac{1}{2}\sin(\omega t)\right) = 3\frac{V_{Ln}}{R_L} = \sqrt{3}\frac{V_{LL}}{R_L}$$



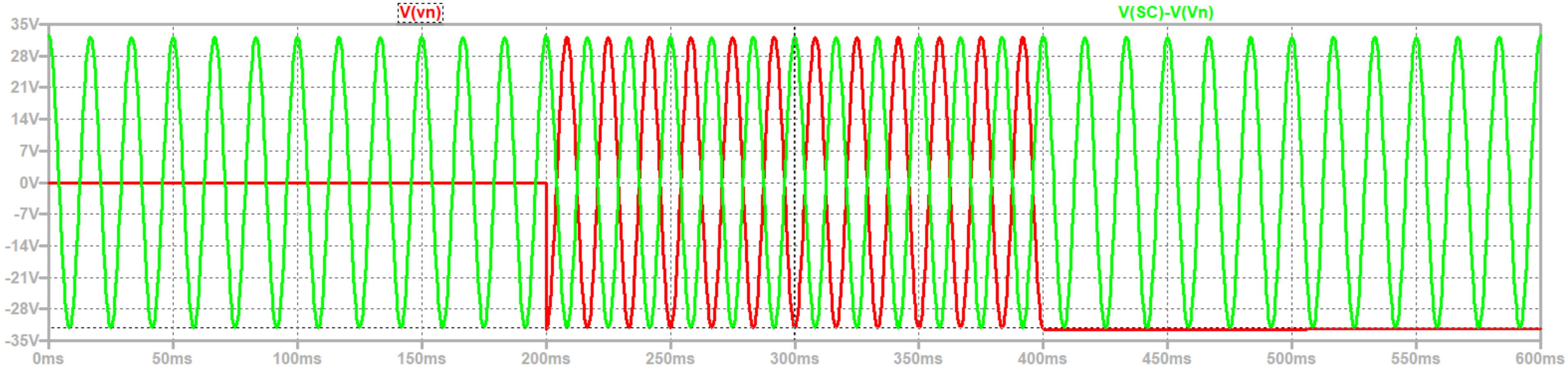
$$I_{a\_calc} = 3\frac{V_{Ln}}{R_L} = 3\frac{32}{300} = .32 \; amps \; peak$$



Line to Line and Line to Neutral Voltages Unchanged during ground fault (Ungrounded Delta-Delta)

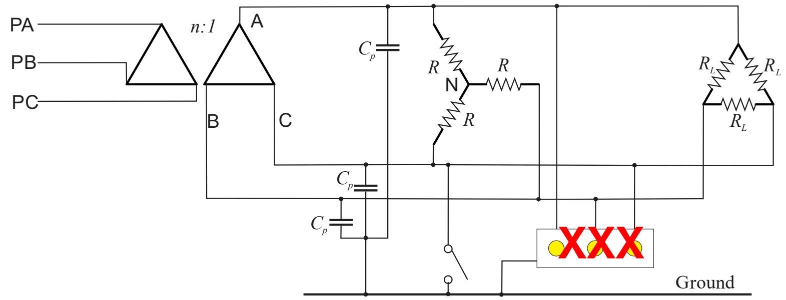


Neutral to ground voltage is negative line to neutral voltage during a ground fault;
After fault clears, may have dc offset (Ungrounded Delta-Delta)



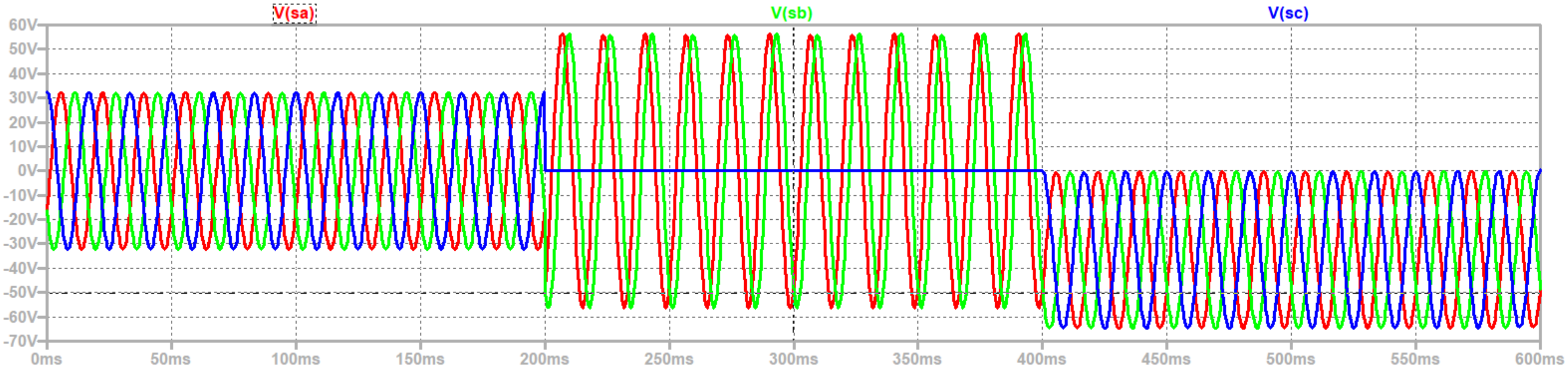
Ground Fault

Note: Ground Lamps not connected to ground



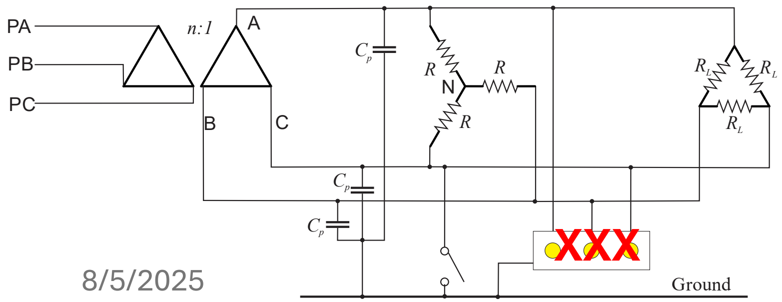
Phase to Ground Voltages (Ungrounded Delta-Delta)

During Fault, line to ground voltages of unfaulted phases equal line to line voltages



Ground Fault

Trapped Charge on capacitors may result in dc offset



Note: Ground Lamps not connected to ground

Ground Fault Currents (Common Mode)

- The neutral resistors and load resistors do not contribute to ground current

The current through the capacitors are

$$I_{CpA} = -V_{CA}\omega C_P \hat{j} = V_{ll}\angle 180 \times \omega C_P \hat{j}$$

$$I_{CpB} = V_{BC}\omega C_P \, \hat{j} \; = V_{ll}\angle 120 \times \; \omega C_P \hat{j}$$

$$I_{CpC} = 0$$

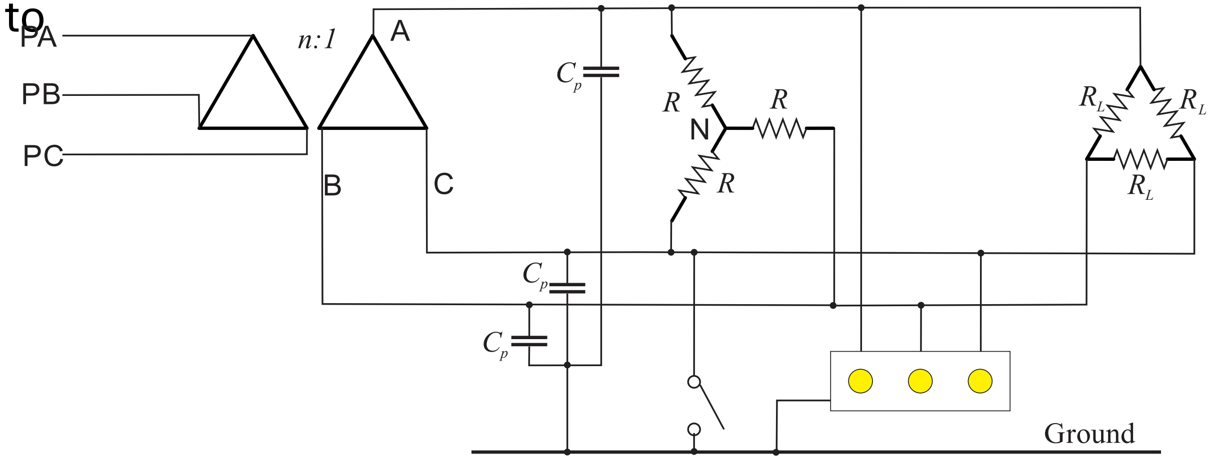
$$I_{Ccm} = I_{CpA} + I_{CpB} + I_{CpC}$$

$$I_{Ccm} = (V_{ll}\angle 180 + V_{ll}\angle 120) \times \; \omega C_P \hat{j}$$

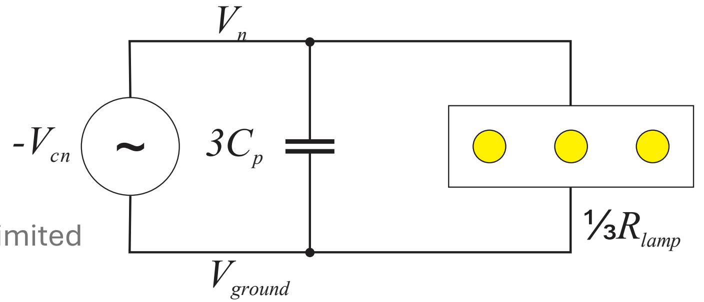
$$I_{Ccm} = \left(\sqrt{3} V_{ll}\angle 150\right) \times \; \omega C_P \hat{j}$$

$$|I_{Ccm}| = V_{ln}\omega 3 C_P$$

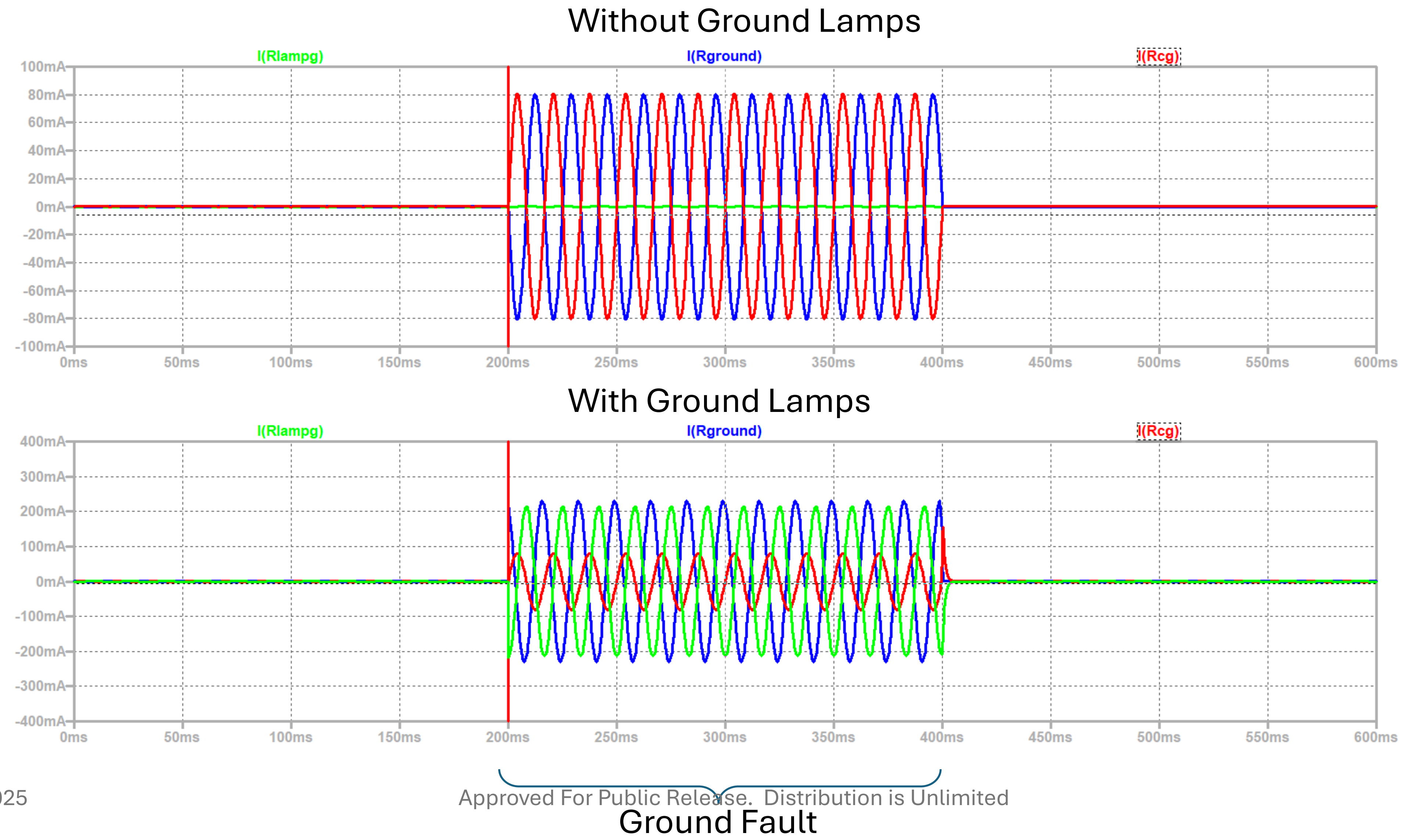
$$\left|I_{Rlamp\_cm}\right| = V_{ln}\frac{3}{R_{lamp}}$$



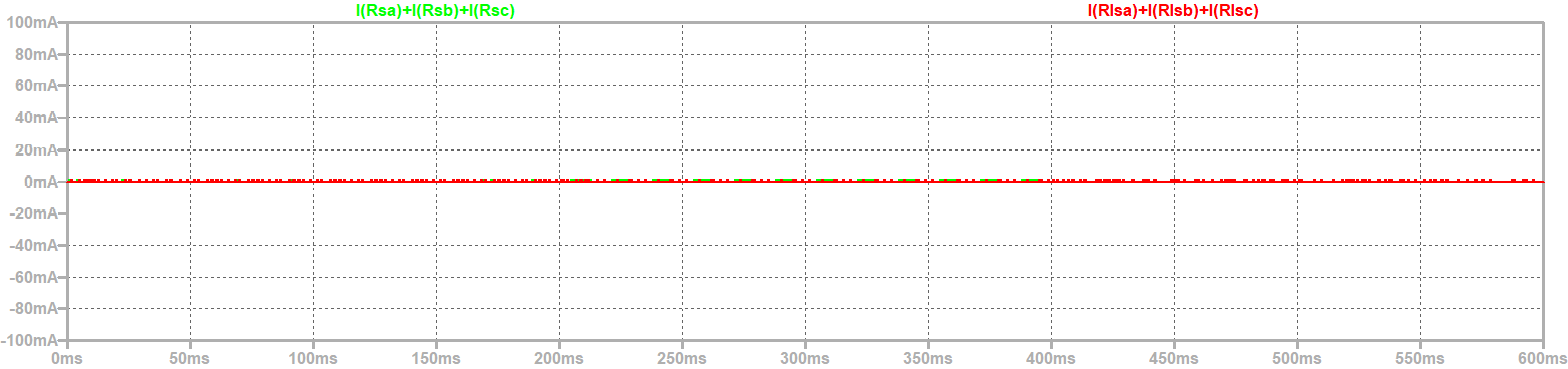
Common Mode Equivalent Circuit

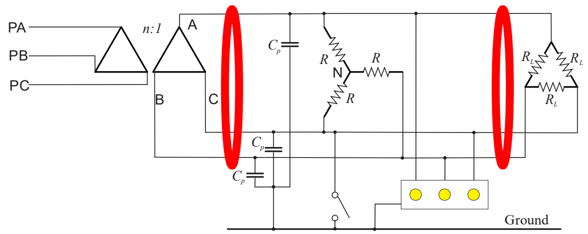


Common Mode Currents to ground (Ground Fault in ungrounded Delta-Delta)



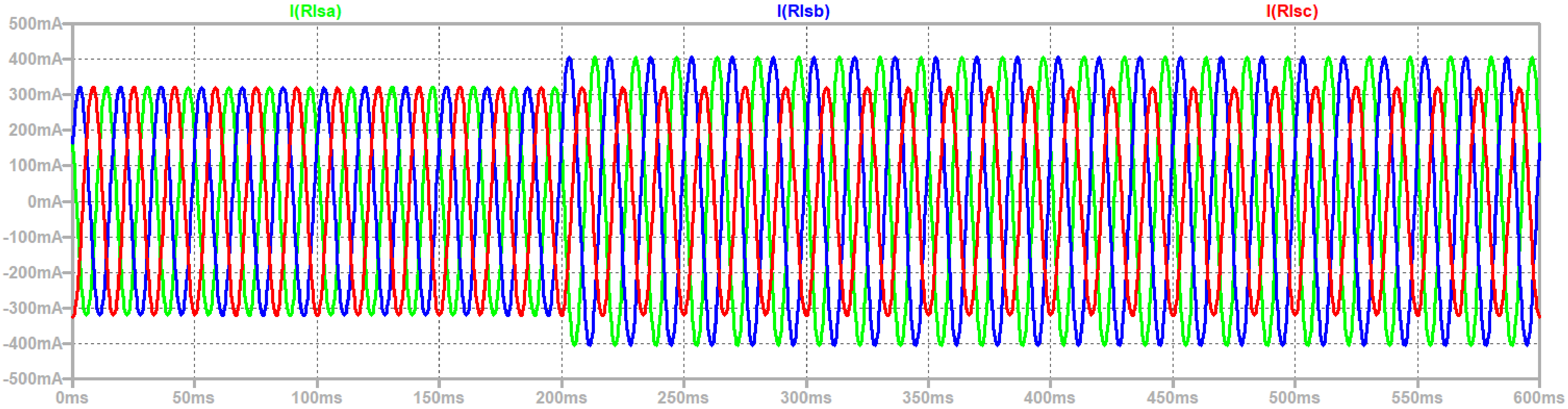
Common Mode Currents in bus at secondary of Transformer and at Load (Ground Fault in ungrounded Delta-Delta)





Kirchhoff Current Law (KCL) implies Comm mode current out of transformer is zero
KCL also implies Common mode current into delta load resistors is zero

Line currents into the delta load not balanced with unbalanced load (Ungrounded Delta-Delta)



Balanced Load | Unbalanced Load

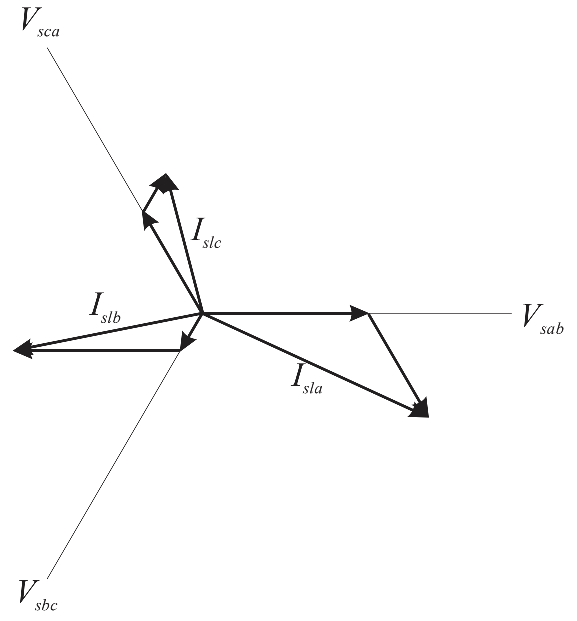
Calculating the secondary line currents into delta load (Ungrounded Delta-Delta)

- If we generalize such that all the load resistors have different values, the line currents into the delta load are given by:

$$I_{sla} = \frac{V_{sab}}{R_{Lab}} - \frac{V_{sca}}{R_{Lca}}$$

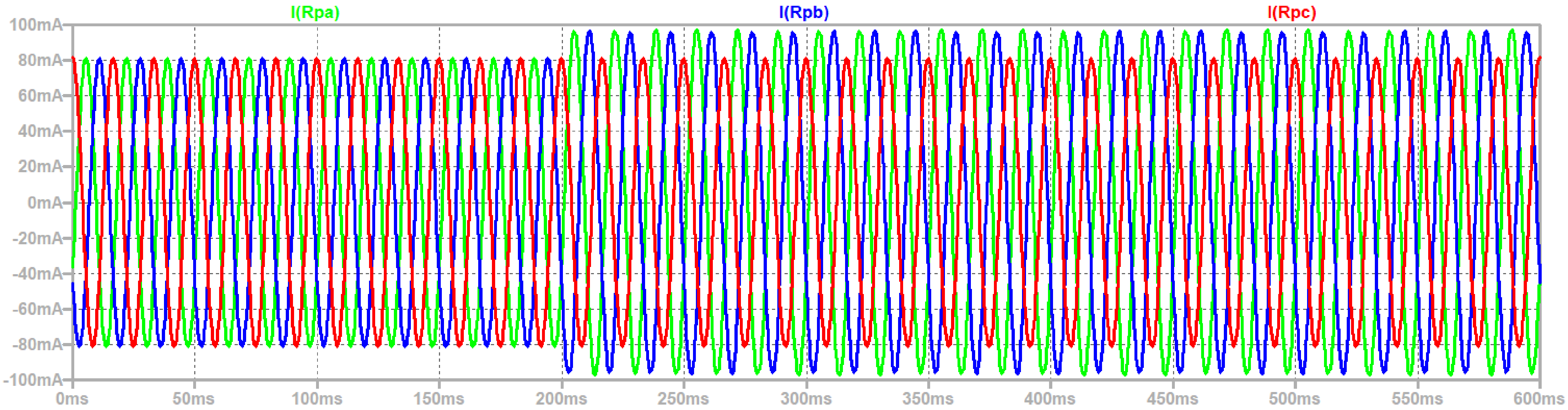
$$I_{slb} = -\frac{V_{sab}}{R_{Lab}} + \frac{V_{sbc}}{R_{Lbc}}$$

$$I_{slc} = -\frac{V_{sbc}}{R_{Lbc}} + \frac{V_{sca}}{R_{Lca}}$$



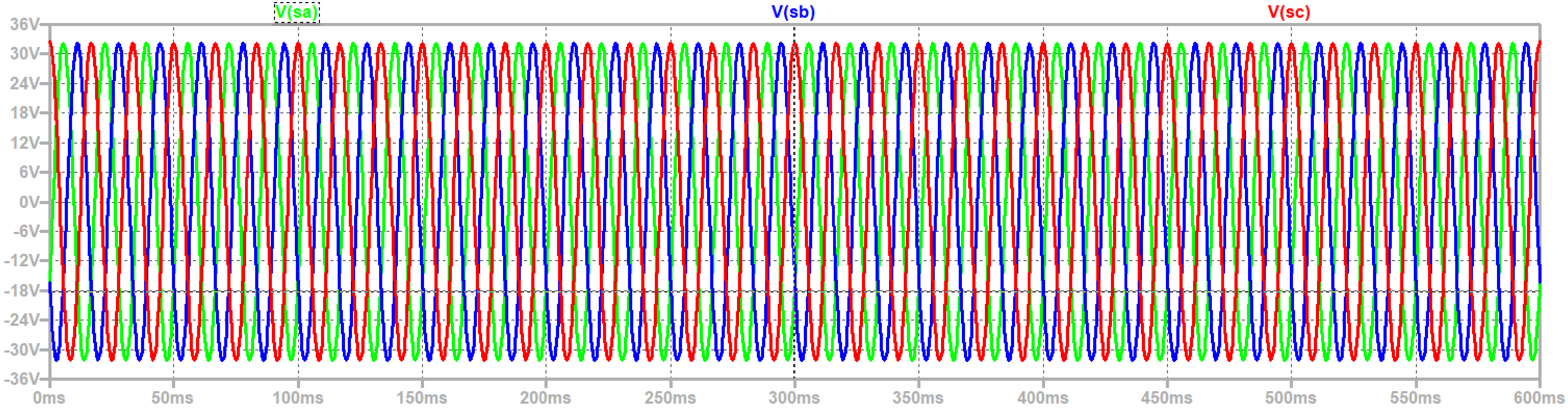
- While the secondary line currents in the delta load resistors sum to zero, they are not necessarily 120° apart from each other.

Line Currents on Primary are also not balanced with unbalanced load (Ungrounded Delta-Delta)



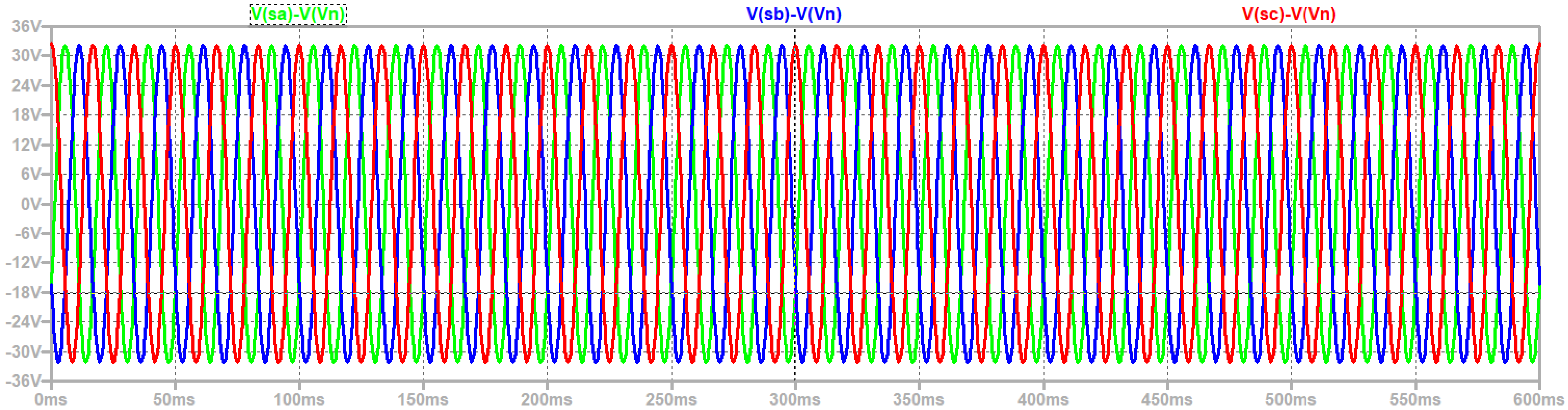
Balanced Load | Unbalanced Load

Phase to Ground Voltages are not impacted by unbalanced load (Ungrounded Delta-Delta)



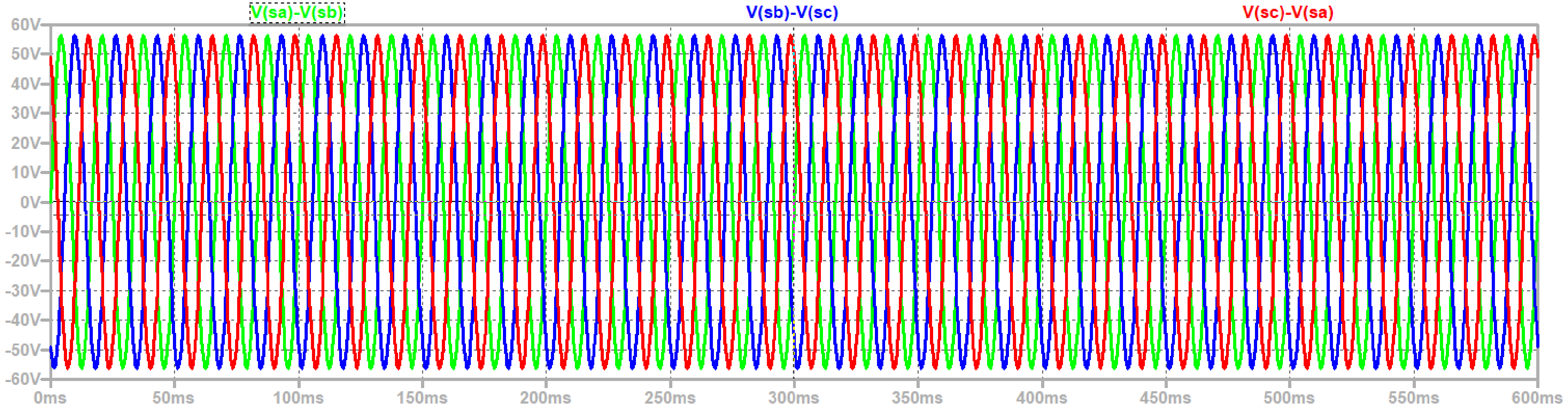
Balanced Load | Unbalanced Load

Phase to neutral voltages are not impacted by unbalanced load (Ungrounded Delta-Delta)



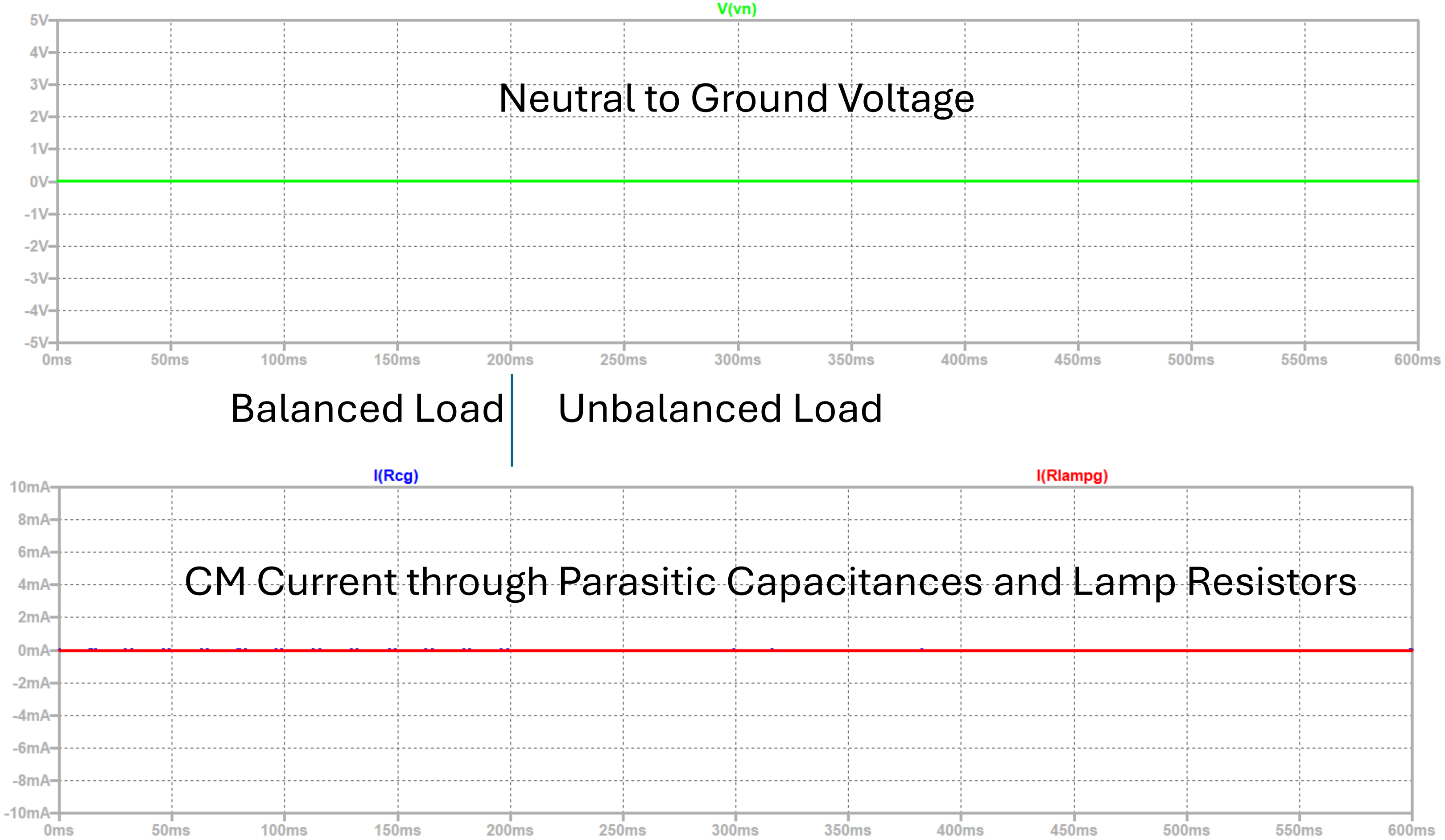
Balanced Load | Unbalanced Load

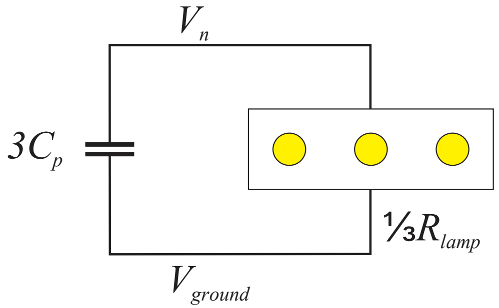
Line to Line voltages are not impacted by unbalanced load (Ungrounded Delta-Delta)



Balanced Load | Unbalanced Load

No neutral to ground voltage (CM Voltage) or CM Current through Parasitic Capacitances or lamp resistances (Ungrounded Delta-Delta)

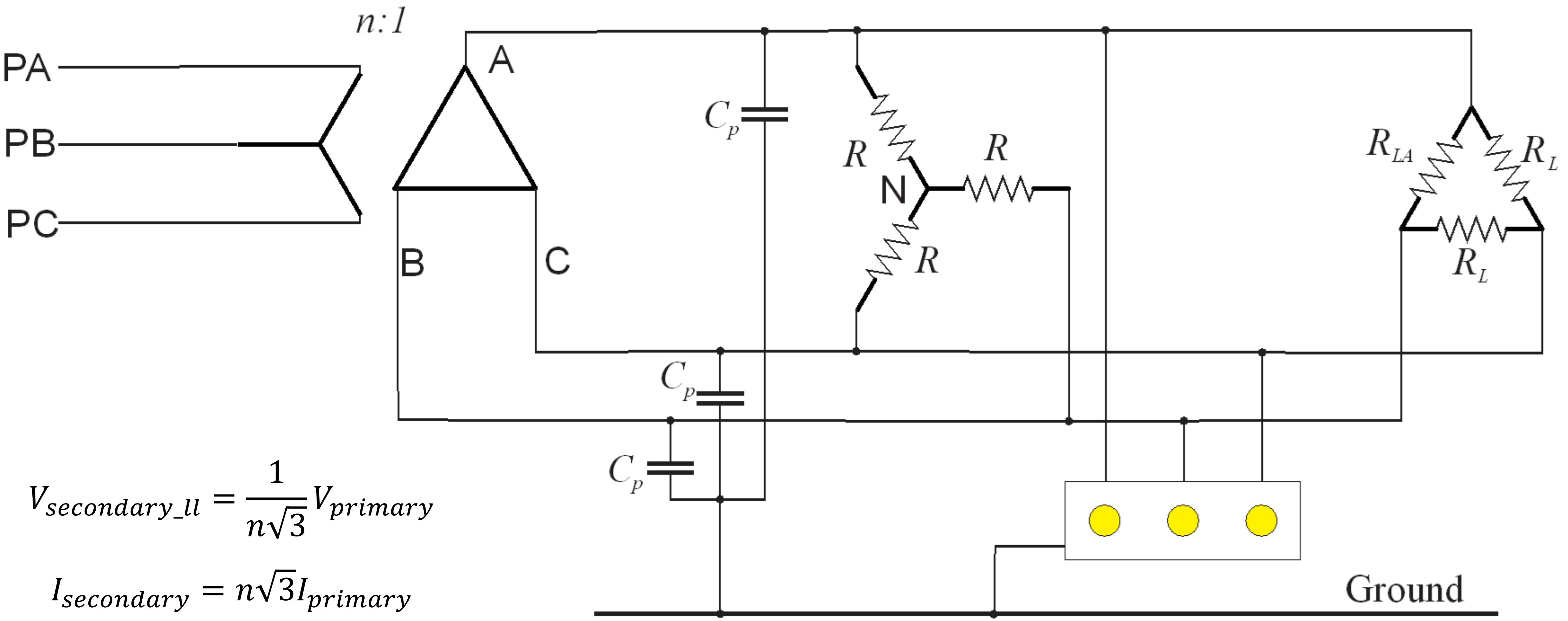




Delta-Delta Summary

- Effective turns ratio: n
- Cable connected to delta winding cannot have common-mode current.
  - If one ignores parasitic capacitances on the delta winding
- Unbalanced delta loads impact magnitude of phase currents and their phase angles.
  - Phase currents still add up to zero.
- Ground Fault on ungrounded system:
  - During ground fault:
    - Neutral to ground voltage is negative line to neutral voltage.
    - Line to Line voltage is not impacted – Line to neutral voltage is not impacted
    - On unfaulted phases, the line to ground voltage is the line-to-line voltage.
    - Fault current determined by parasitic capacitance (fed from unfaulted phases).
  - After fault clears, may have dc offset.

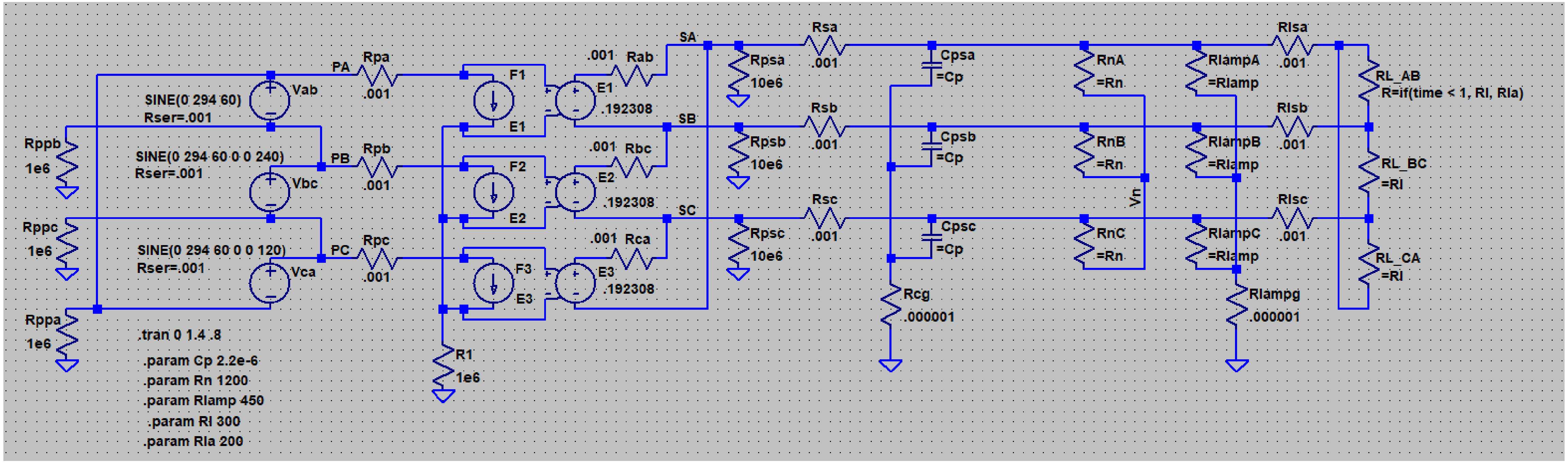
Wye-Delta Transformers
Ungrounded secondary



$$V_{secondary\_ll} = \frac{1}{n\sqrt{3}} V_{primary}$$

$$I_{secondary} = n\sqrt{3} I_{primary}$$

Simulation Model – Wye-Delta



20240831 experiment 4.asc

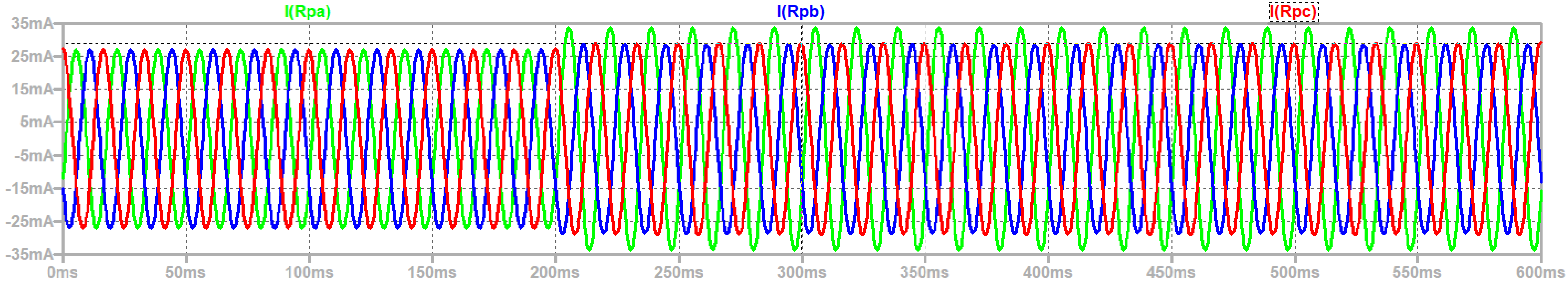
LTspice

Ground Fault response for Wye-Delta and Delta-Delta the same for ground fault in ungrounded system.

- Connection to delta winding cannot have a CM current (Kirchhof’s current law)
- CM Circuit does not depend on the primary winding.

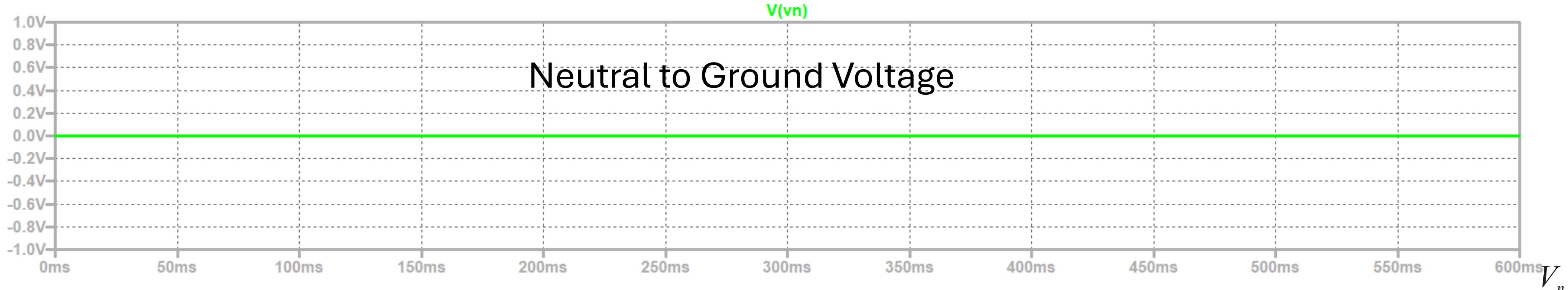
Line currents into the delta load not balanced with unbalanced load (Ungrounded Wye-Delta)

Line Currents on Primary are also not balanced with unbalanced load (Ungrounded Wye-Delta)

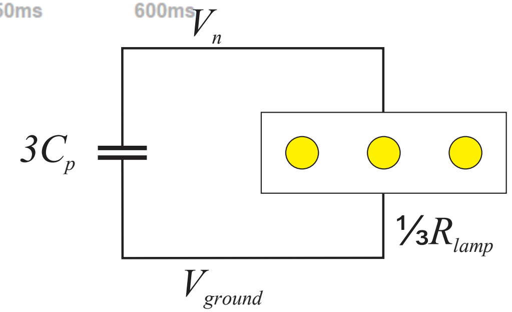


Balanced Load | Unbalanced Load

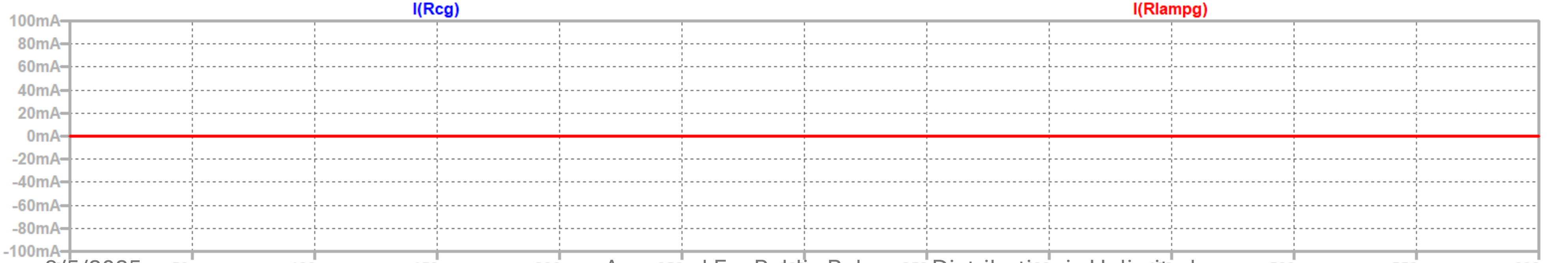
No neutral to ground voltage (CM Voltage) or CM Current through Parasitic Capacitances or lamp resistances due to unbalanced load (Ungrounded Wye-Delta)



Balanced Load | Unbalanced Load



CM Current through Parasitic Capacitances and Lamp Resistors

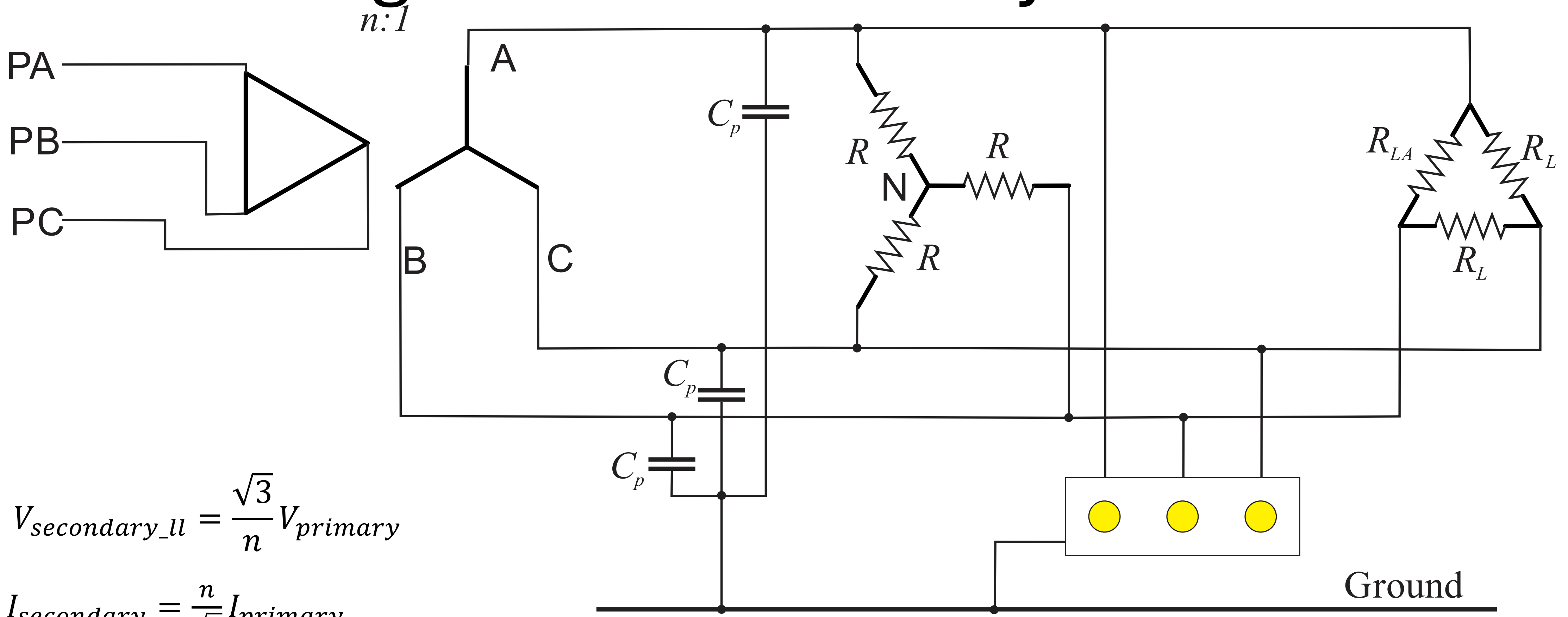


Wye-Delta Summary

- Effective turns ratio: $n\sqrt{3}$ with 30° phase shift.
- Unbalanced delta loads impact magnitude of phase currents and their phase angles.
  - Phase currents still add up to zero.
- Cable connected to delta winding cannot have common-mode current.
  - If one ignores parasitic capacitances on the delta winding.
- If Wye common point is unterminated, cable connected to wye winding cannot have common-mode current
  - If one ignores parasitic capacitances.
- Ground Fault on ungrounded system (delta connection):
  - Same performance as for Delta-Delta system.

Break

Delta-Wye Transformers
3-wire Ungrounded secondary

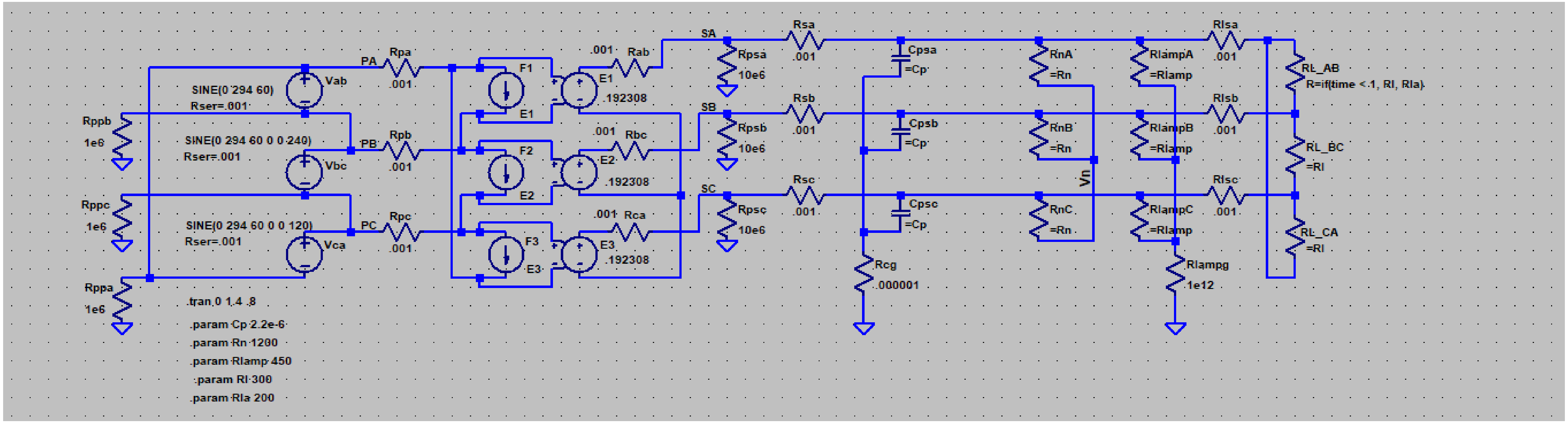


$$V_{secondary\_ll} = \frac{\sqrt{3}}{n} V_{primary}$$

$$I_{secondary} = \frac{n}{\sqrt{3}} I_{primary}$$

(Neglecting losses)

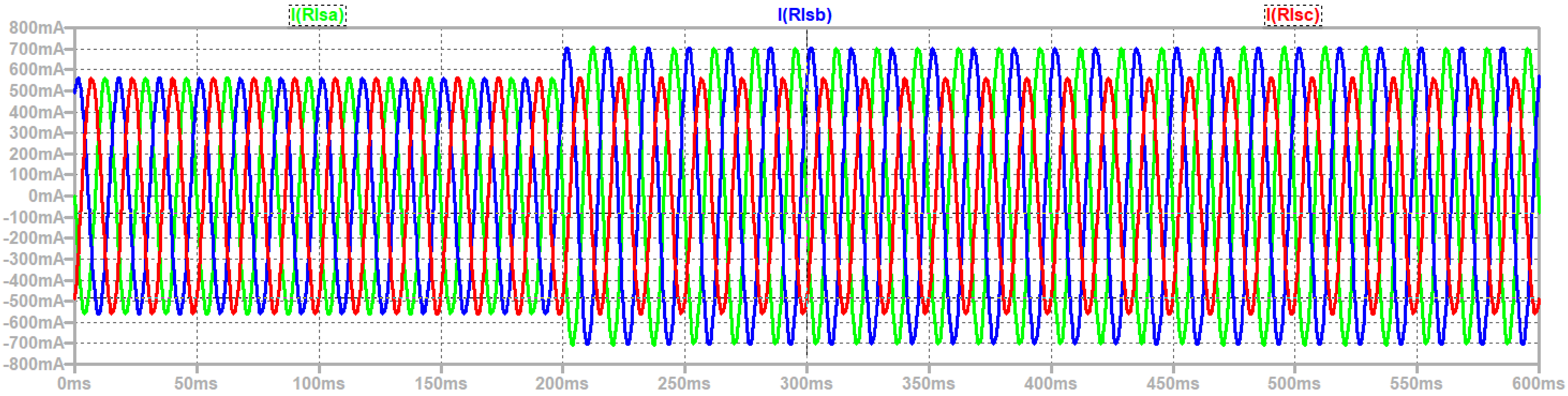
Simulation Model



20240831 experiment 5.asc

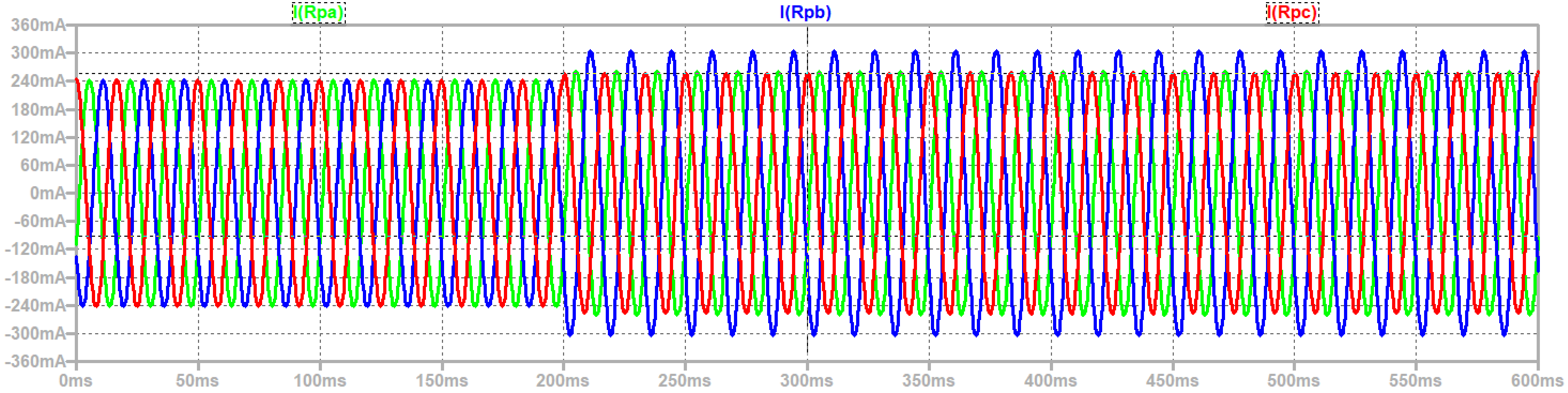
LTspice

Line currents into the delta load not balanced with unbalanced load



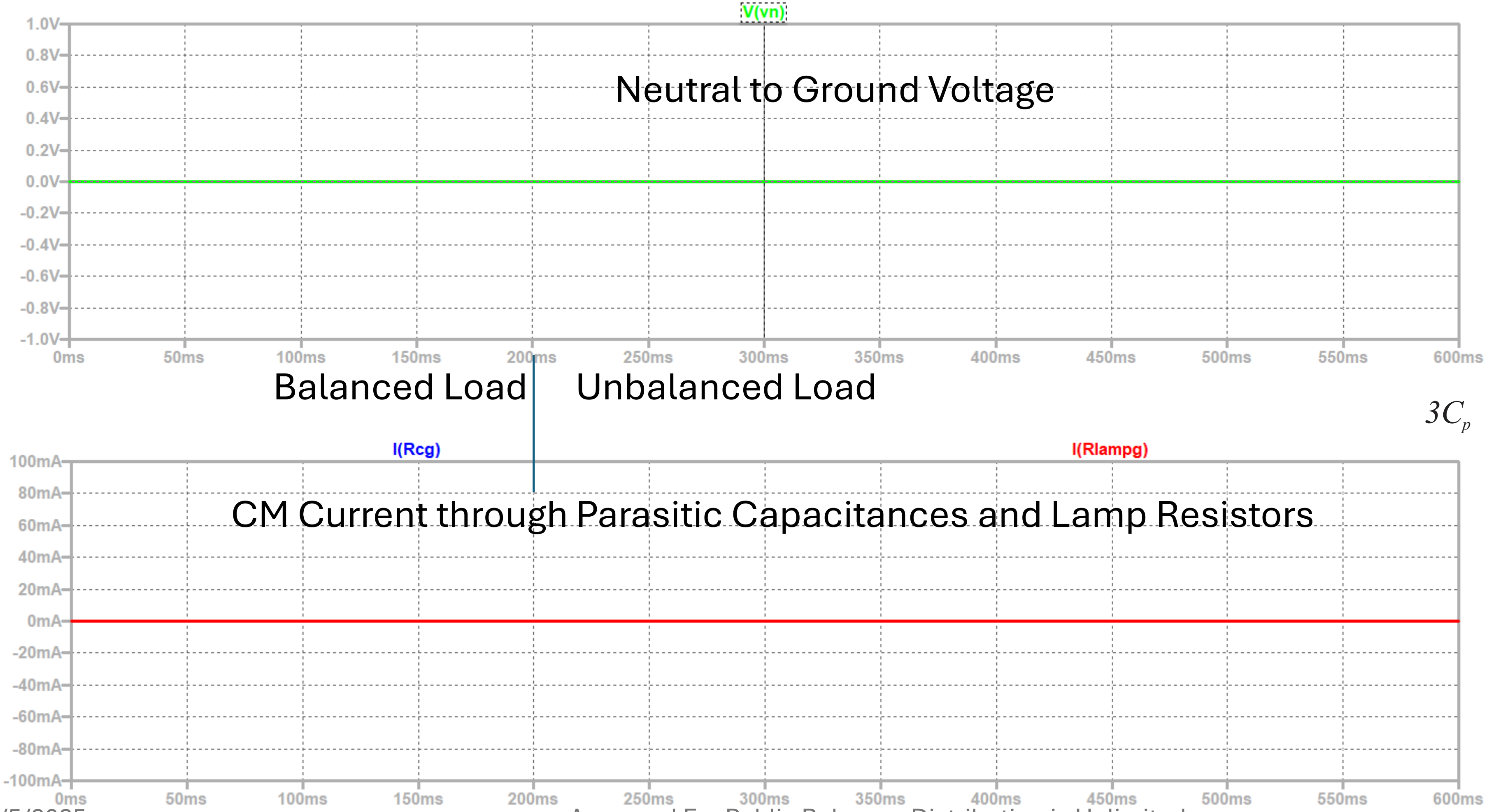
Balanced Load | Unbalanced Load

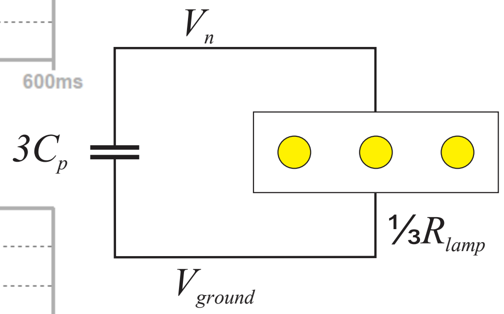
Line Currents on Primary are also not balanced with unbalanced load



Balanced Load | Unbalanced Load

No neutral to ground voltage (CM Voltage) or CM Current through Parasitic Capacitances or lamp resistances





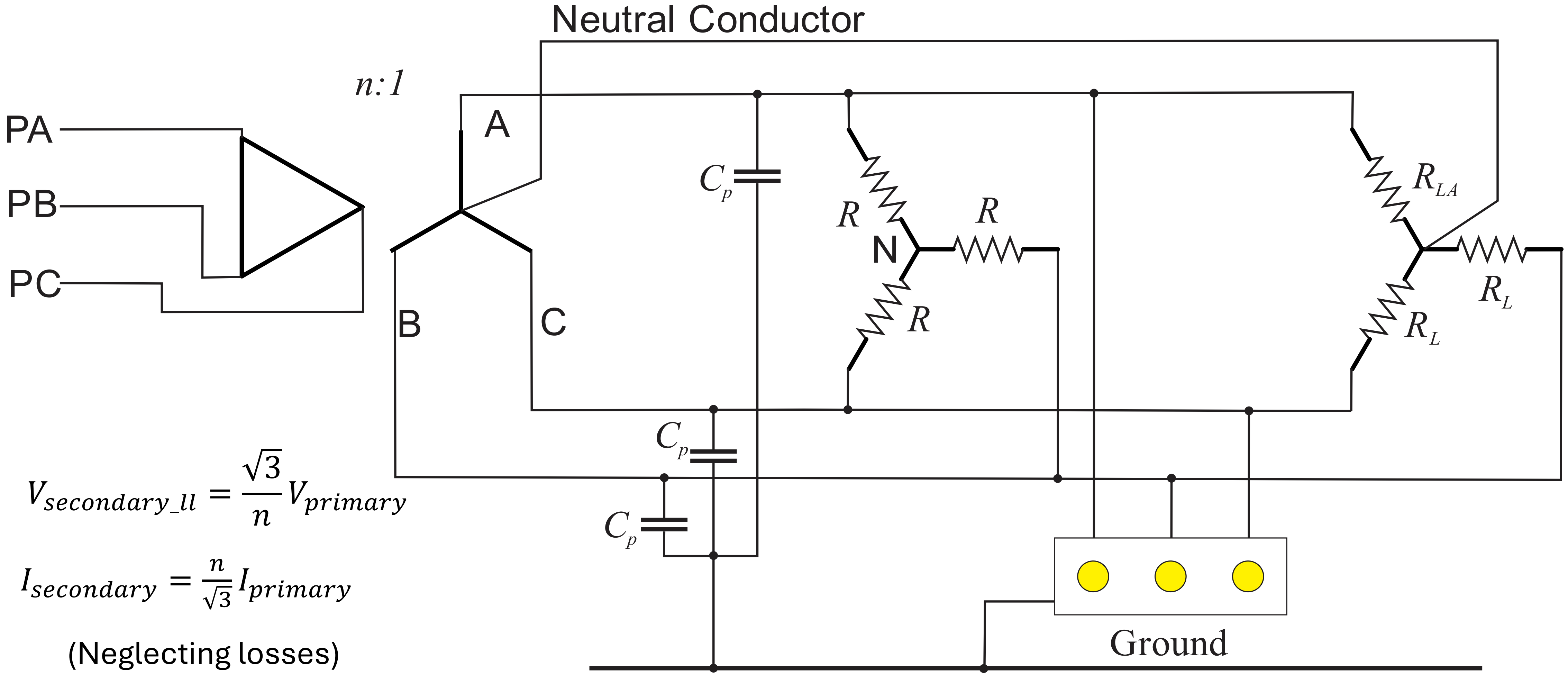
Ground Fault response for Delta-Wye and Delta-Delta the same for ground fault in ungrounded system.

- Connection to wye winding (without unterminated common connection point) cannot have a CM current (Kirchhof's current law)
- CM Circuit does not depend on the primary winding.

Delta-Wye 3-wire Summary

- Effective turns ratio: $\frac{n}{\sqrt{3}}$ with 30° phase shift.
- Unbalanced delta loads impact magnitude of phase currents and their phase angles.
  - Phase currents still add up to zero.
- If Wye common point is unterminated, cable connected to wye winding cannot have common-mode current
  - If one ignores parasitic capacitances on the wye winding.
- Ground Fault on ungrounded system (wye connection):
  - Same performance as for Delta-Delta system.

Delta-Wye Transformers
4-wire Ungrounded secondary



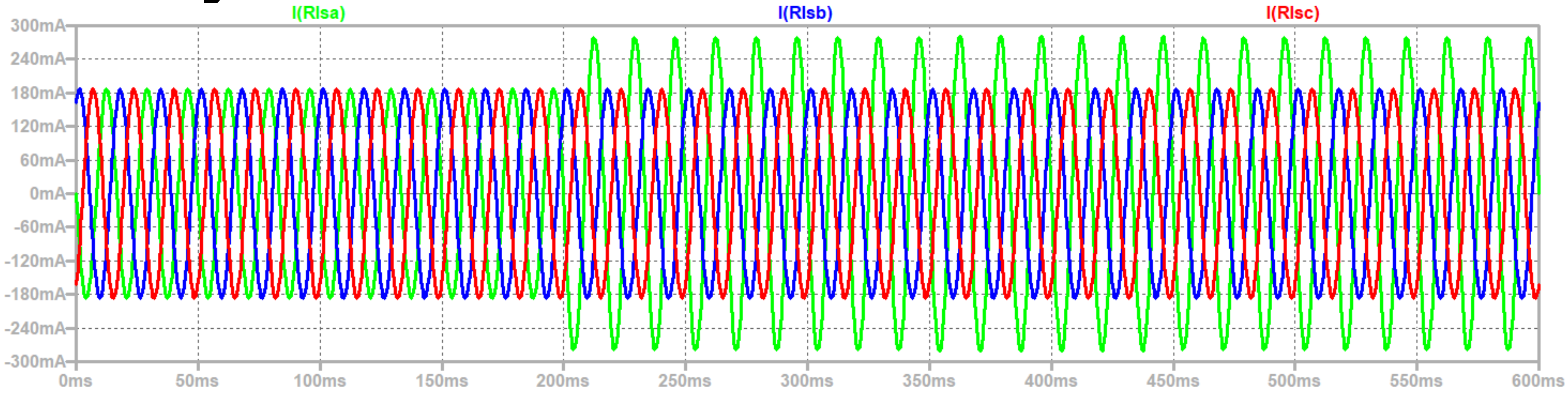
$$V_{secondary\_ll} = \frac{\sqrt{3}}{n} V_{primary}$$

$$I_{secondary} = \frac{n}{\sqrt{3}} I_{primary}$$

(Neglecting losses)

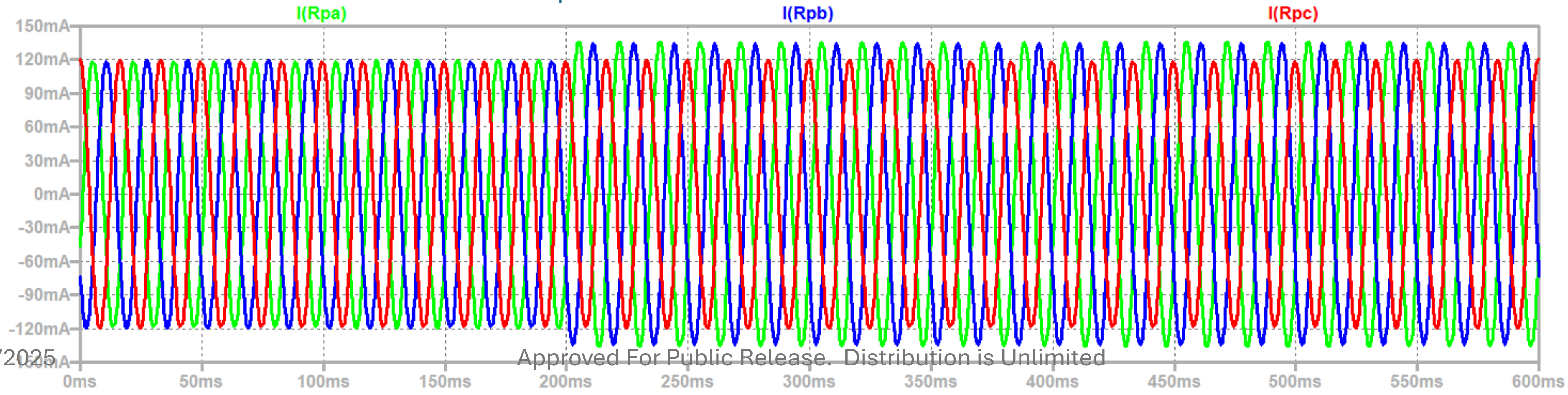
Line currents into the wye load and into primary not balanced with unbalanced load

Current into Wye load



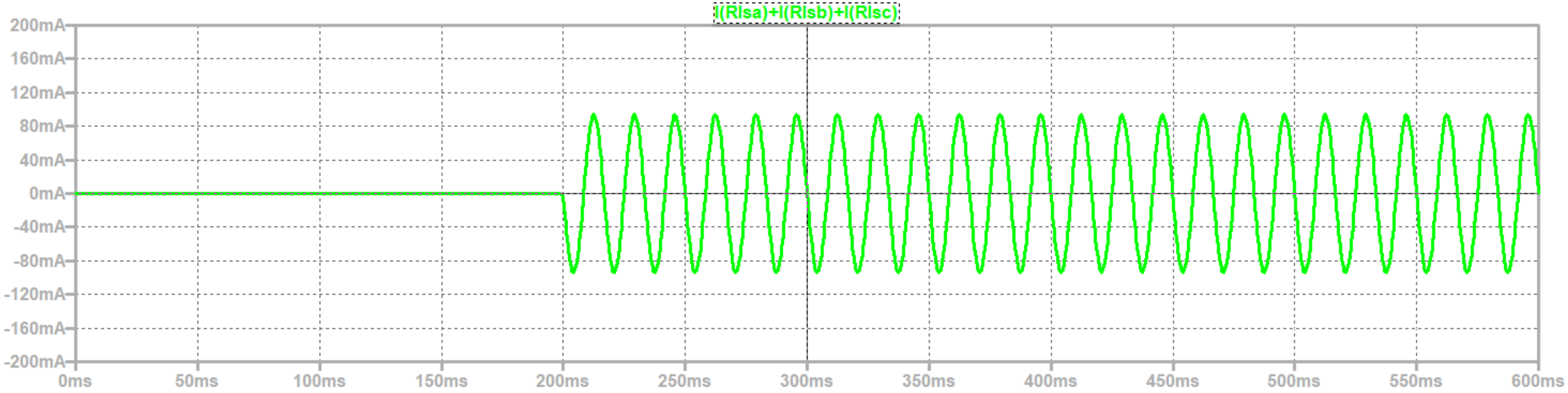
Balanced Load | Unbalanced Load

Current into Primary



Circuit Protection on Primary may not trip with overload of a single conductor on secondary

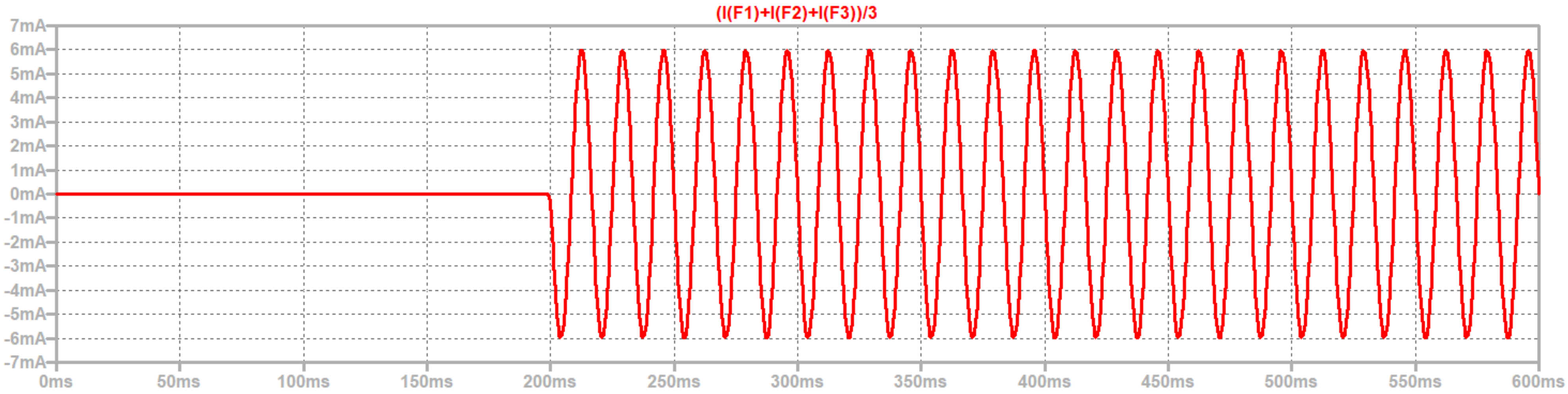
Unbalanced load results in current in the Neutral Conductor



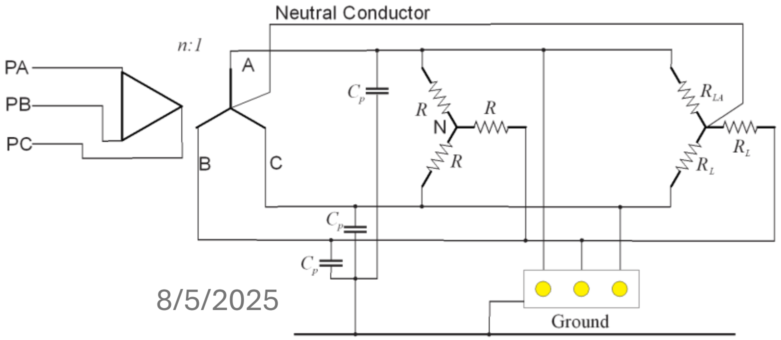
Balanced Load | Unbalanced Load

$$i_{neutral\_conductor} = \frac{\frac{98}{1.73}}{600} = 94.3 \text{ mA peak}$$

Unbalanced load results in circulating current in the delta windings



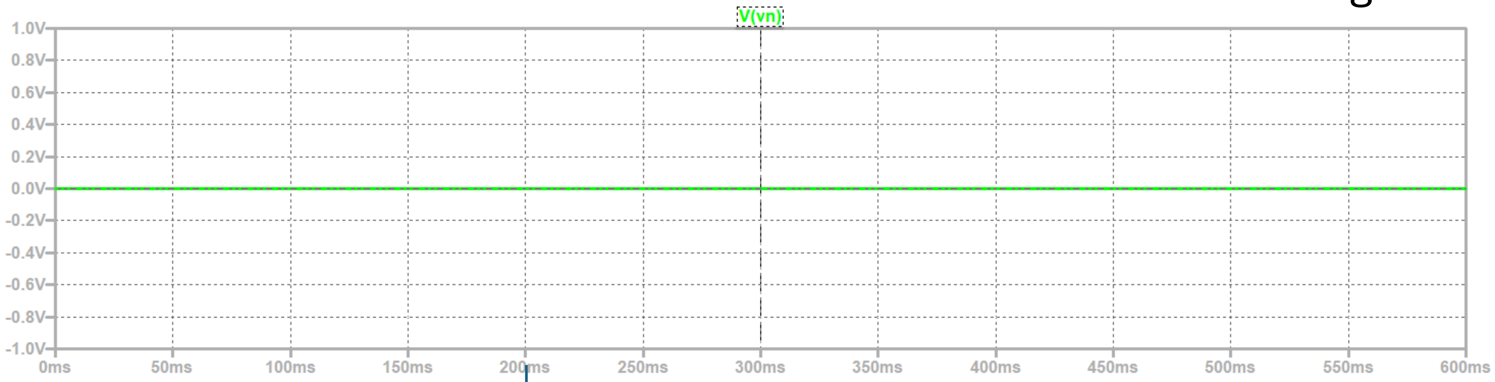
Balanced Load | Unbalanced Load



$$i_{circulatiing} = \frac{I_{secondary\_neutral}}{3n} = \frac{94.3}{3\times 5.2} = 6.0 \text{ mA peak}$$

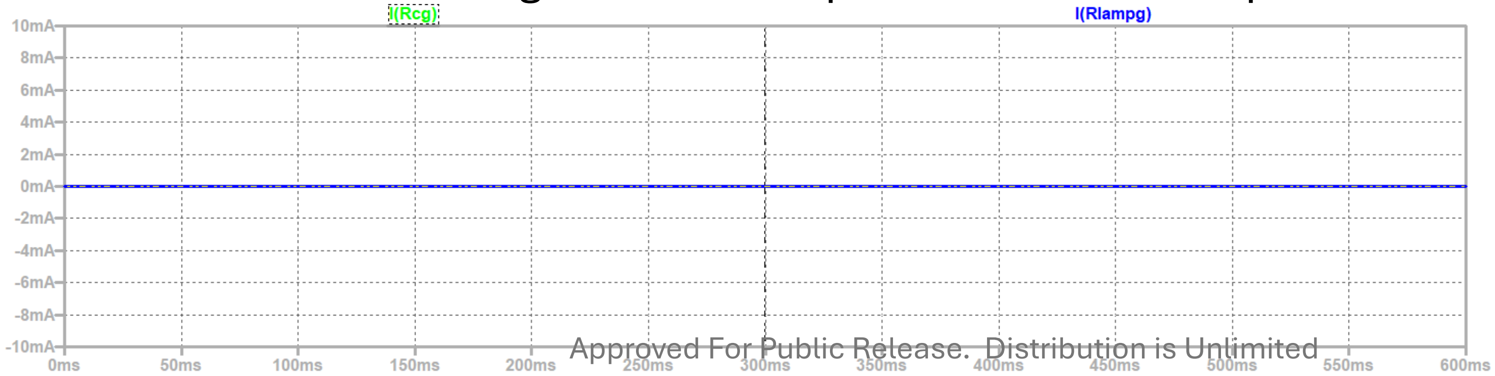
No neutral to ground voltage (CM Voltage) or CM Current through Parasitic Capacitances or lamp resistances

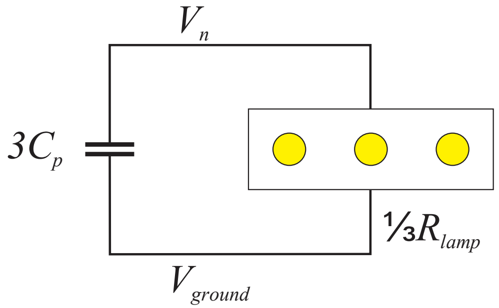
Neutral to Ground Voltage



Balanced Load | Unbalanced Load

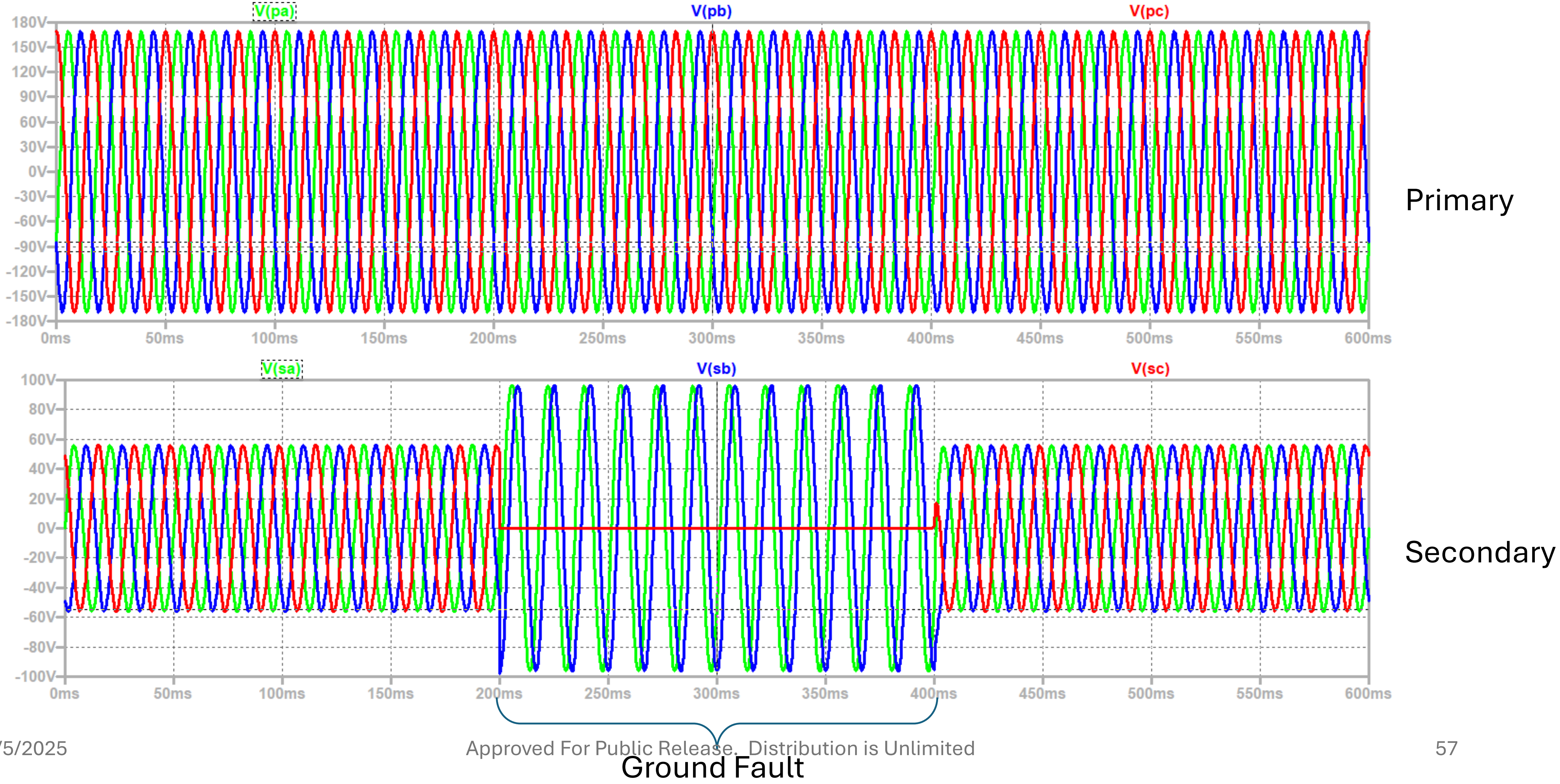
CM Current through Parasitic Capacitances and Lamp Resistors





Line to ground insulation on secondary should be rated for line-to-line voltage

Line to Ground Voltages – Ground Fault

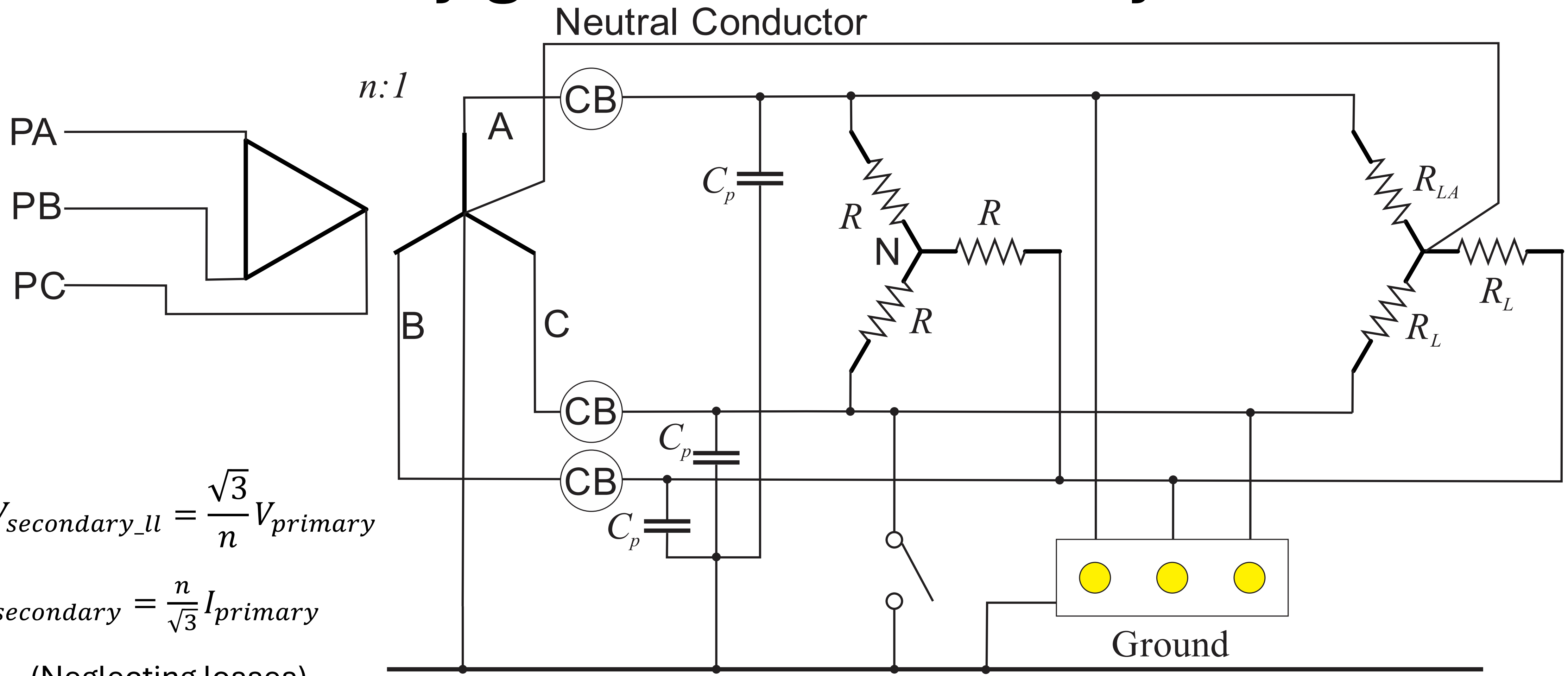


Approved For Public Release. Distribution is Unlimited

Delta-Wye 4-wire Summary (ungrounded)

- Effective turns ratio: $\frac{n}{\sqrt{3}}$ with 30° phase shift.
- Unbalanced delta loads impact magnitude of phase currents and their phase angles.
  - Phase currents DO NOT add to zero: current in neutral conductor
- Three phase conductors connected to wye winding can have a common-mode current
  - Equal to the current in the neutral conductor
- Common-mode current in wye-winding transformed to a circulating current in the delta winding
  - No common-mode current in the cable connected to the delta winding
- Ground Fault on ungrounded system (wye connection):
  - Same performance as for Delta-Delta system.

Delta-Wye Transformers
4-wire solidly grounded secondary



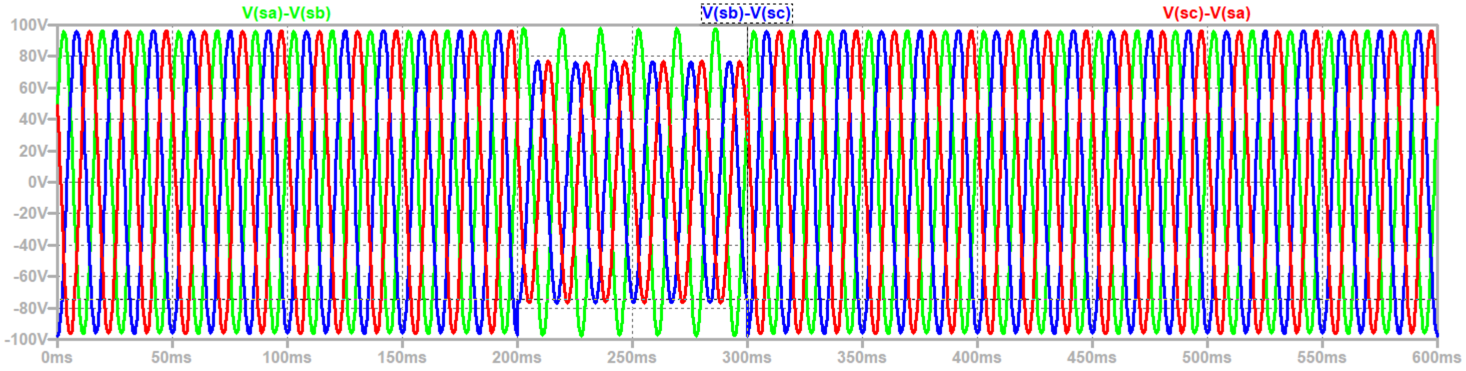
$$V_{secondary\_ll} = \frac{\sqrt{3}}{n} V_{primary}$$

$$I_{secondary} = \frac{n}{\sqrt{3}} I_{primary}$$

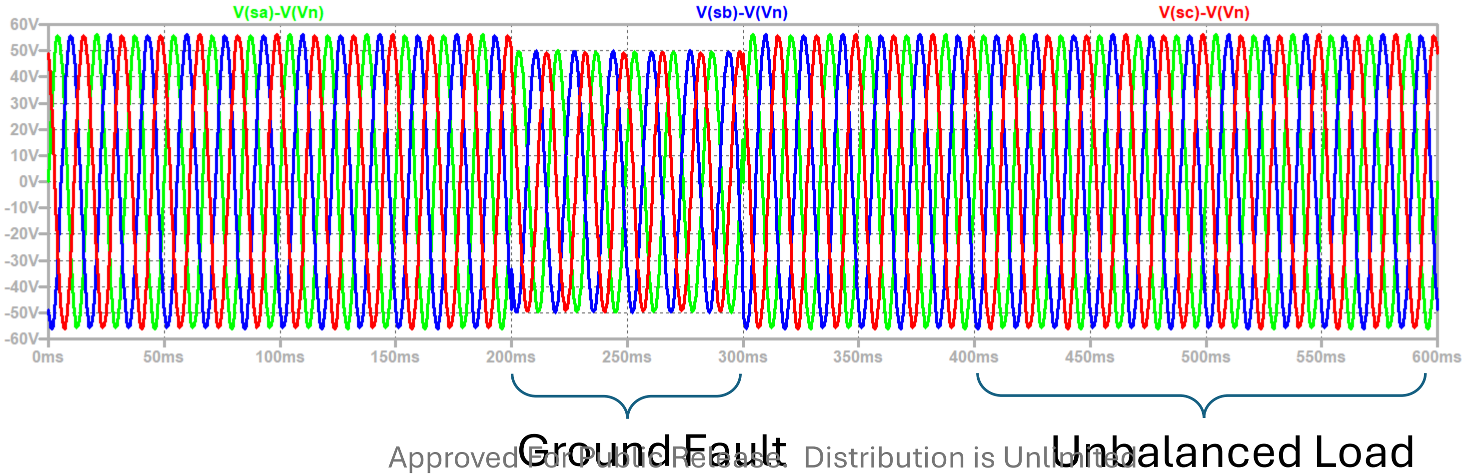
(Neglecting losses)

Steady State Voltages – Ground Fault / Unbalanced Load

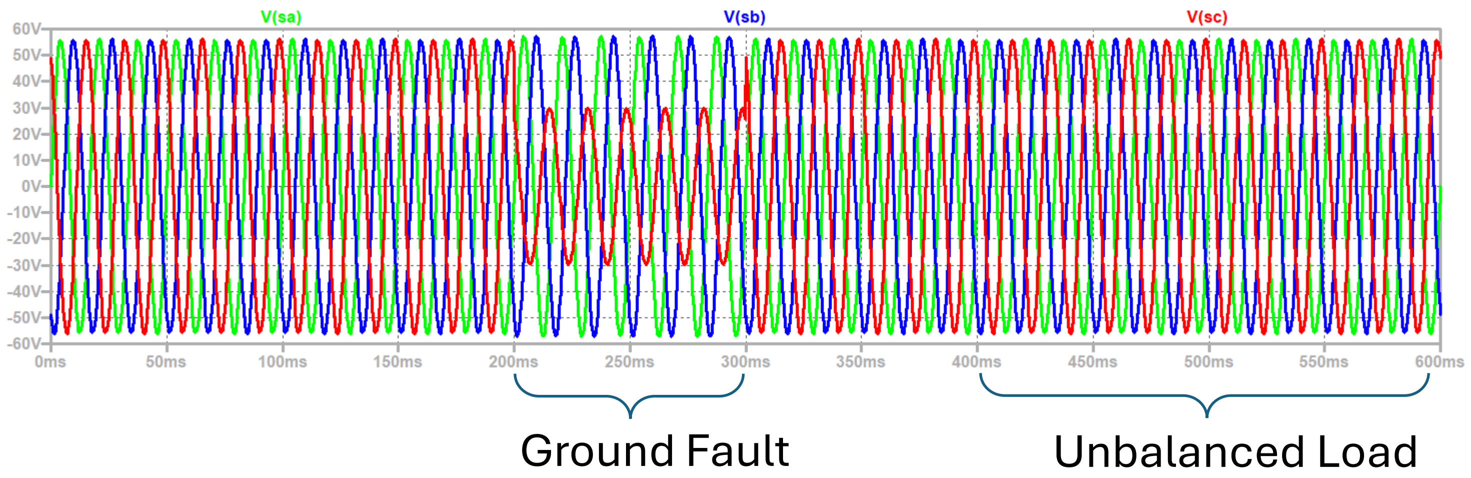
$$V_{secondary\_ll} = \frac{\sqrt{3}}{n} V_{primary} = \frac{\sqrt{3}}{5.2} 208 = 69.3\ volts\ rms\ \ (98\ V\ peak)$$



Secondary Line to Neutral Voltage: Peak = 56.6 V volts

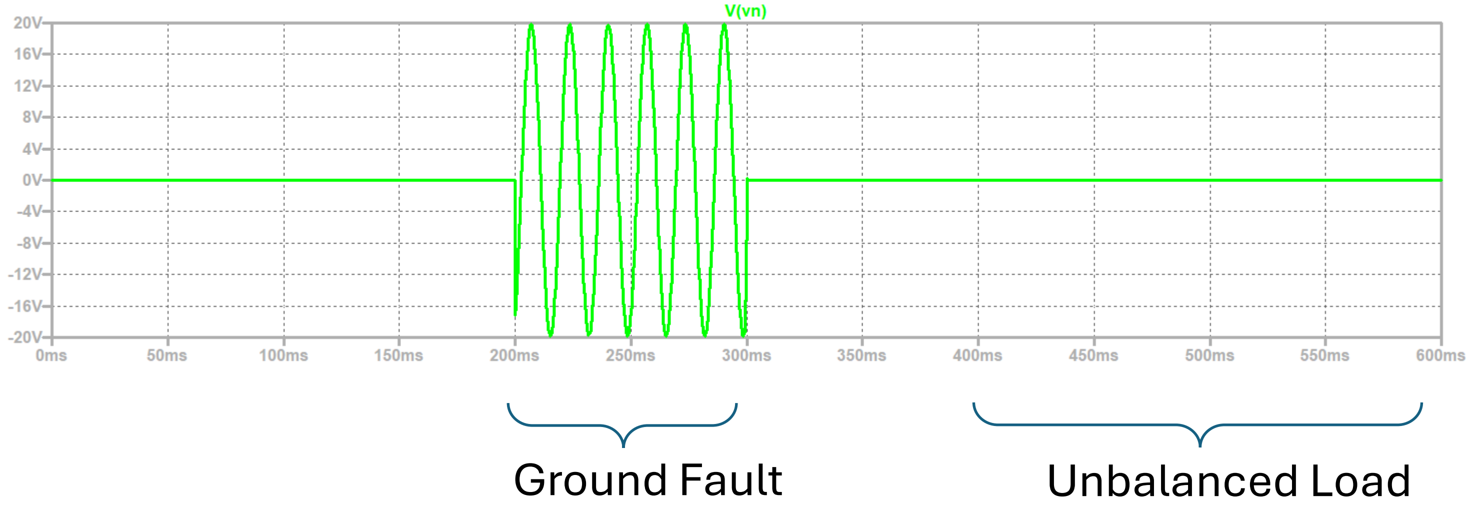


Steady State line to ground Voltages – Ground Fault / Unbalanced Load

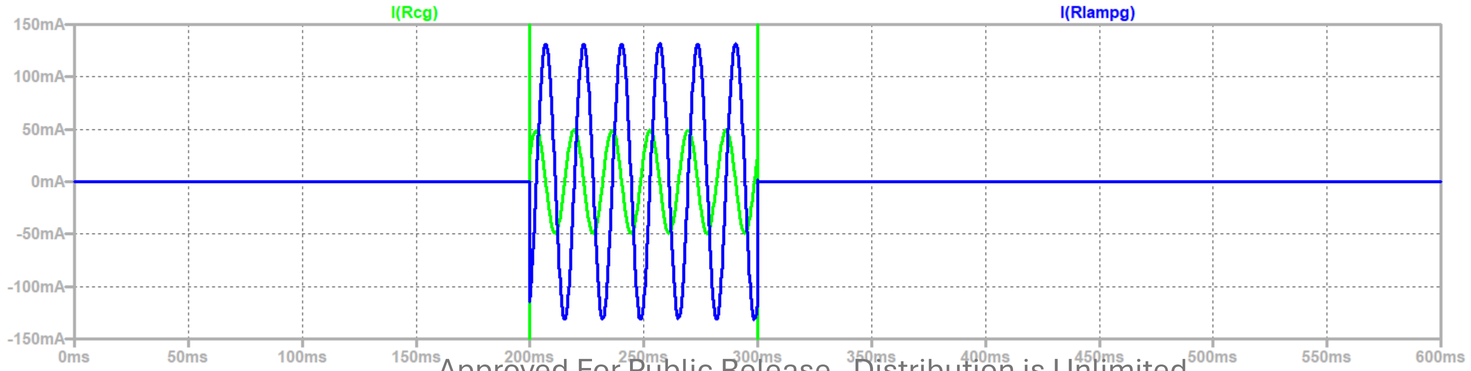


Neutral to ground voltage (CM Voltage) and CM Current through Parasitic Capacitances or lamp resistances

Neutral to Ground Voltage

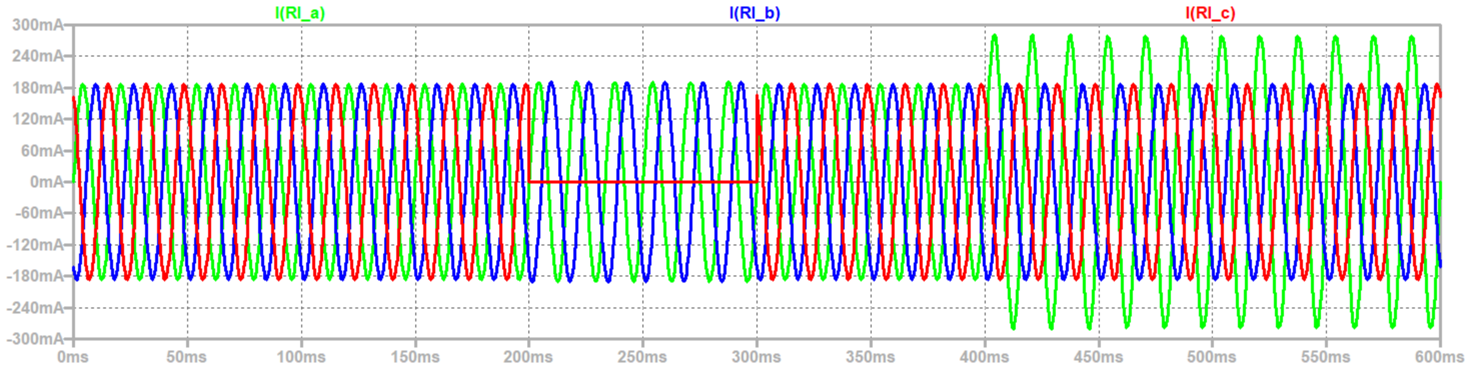


CM Current through Parasitic Capacitances and Lamp Resistors



Line currents into the wye load and into primary not balanced with unbalanced load

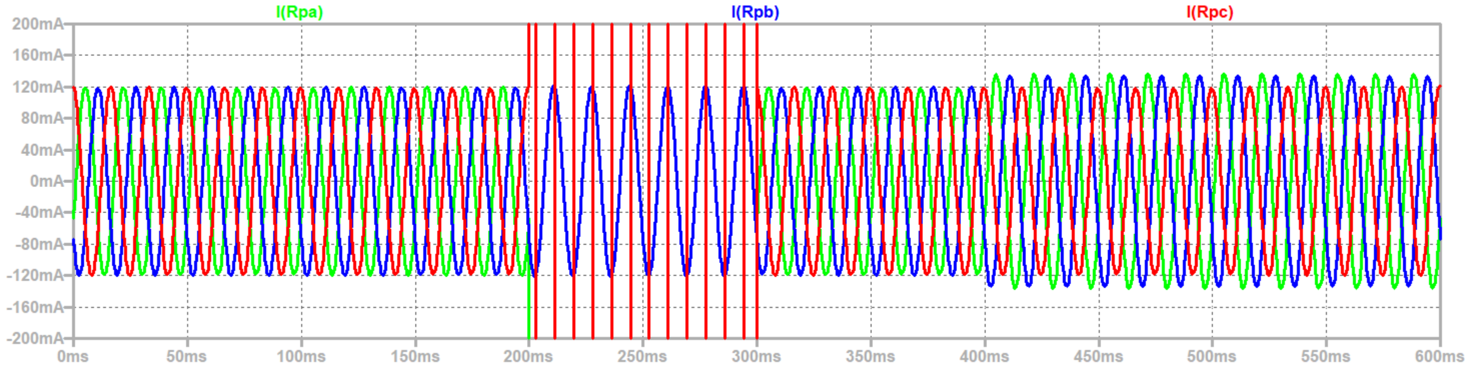
Current into Wye load



Ground Fault

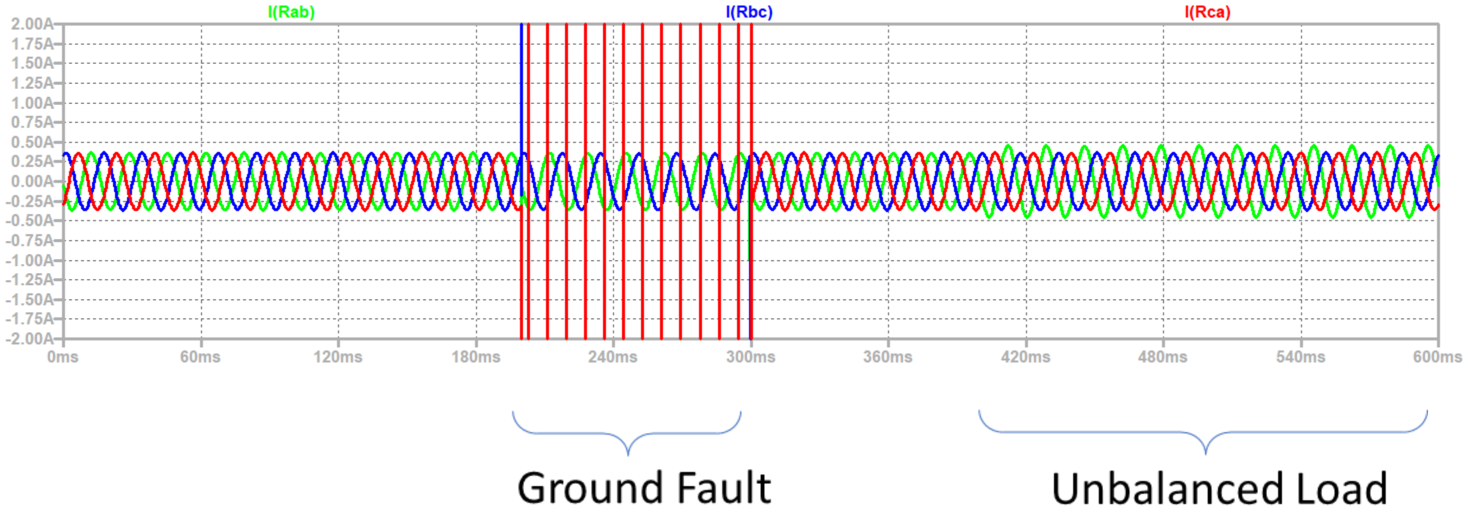
Unbalanced Load

Current into Primary



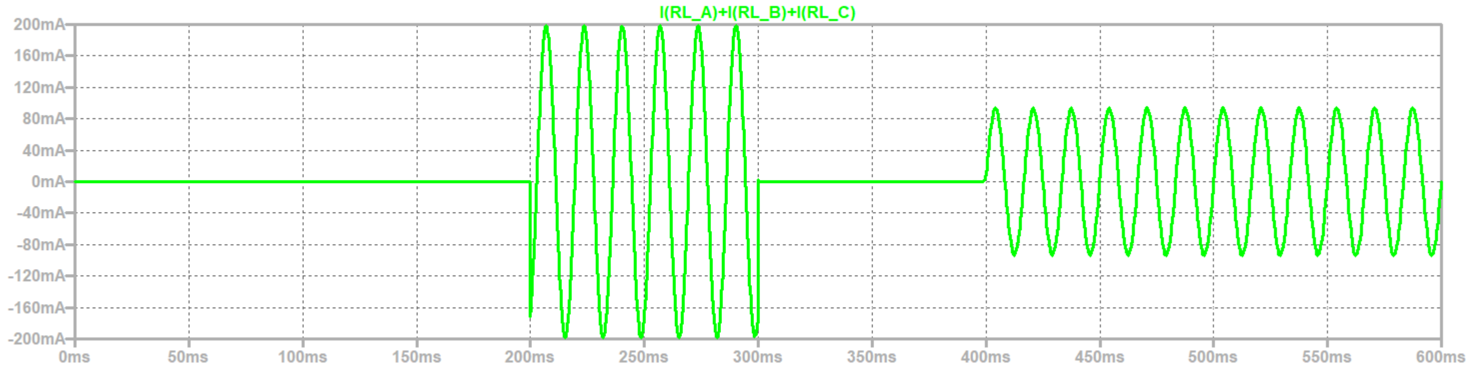
Large currents during ground fault will activate over-current protection

Line Currents out of Transformer



Large currents during ground fault will activate over-current protection

Unbalanced load results in current in the Neutral Conductor



Ground Fault

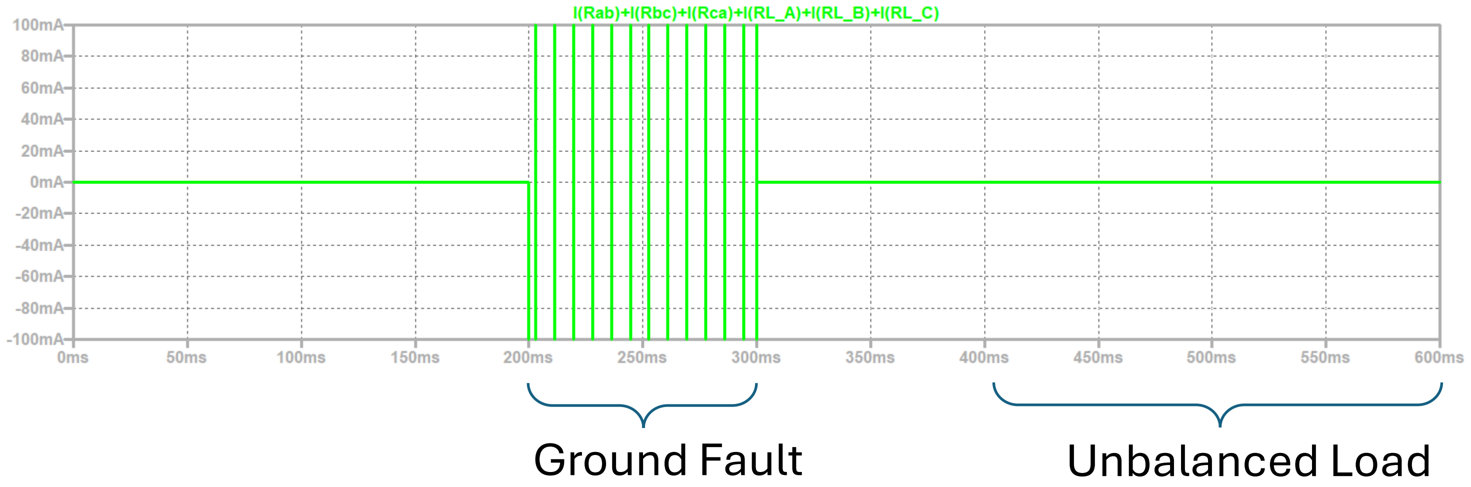
Unbalanced Load

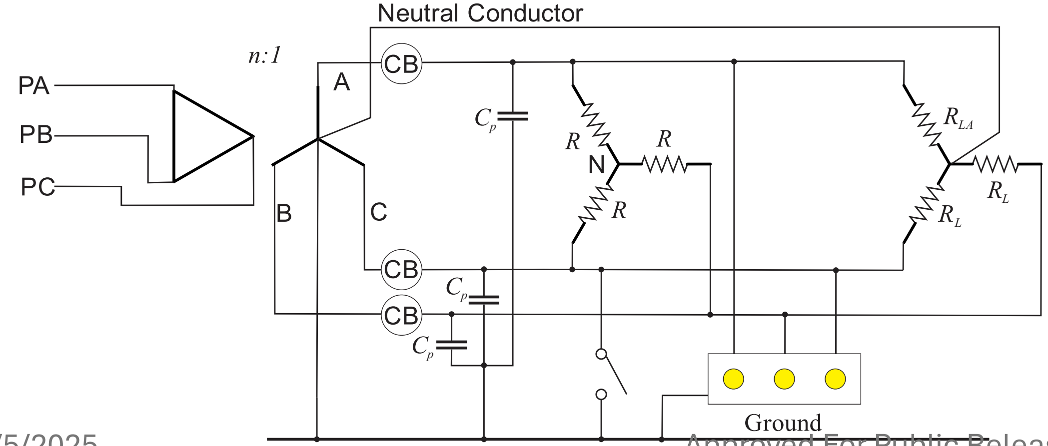
For unbalanced loads

$$i_{neutral\_conductor} = \frac{\frac{98}{1.73}}{600} = 94.3 \text{ mA peak}$$

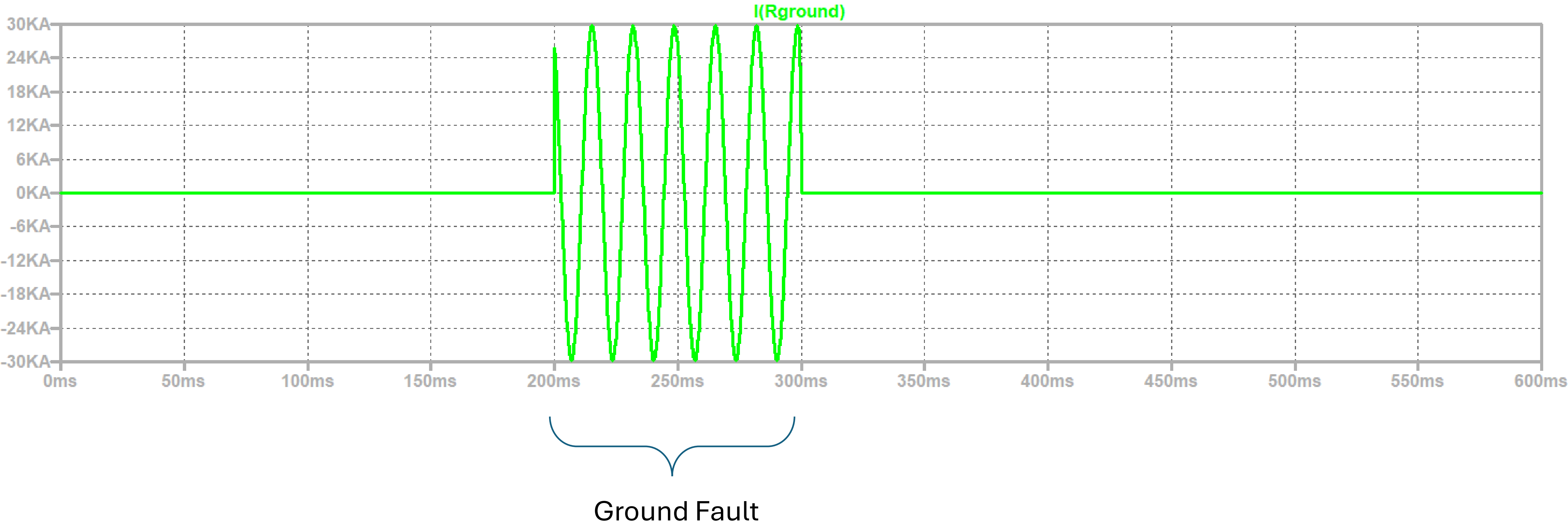
Current through Transformer Ground Connection

No current through the ground connection during an unbalanced load





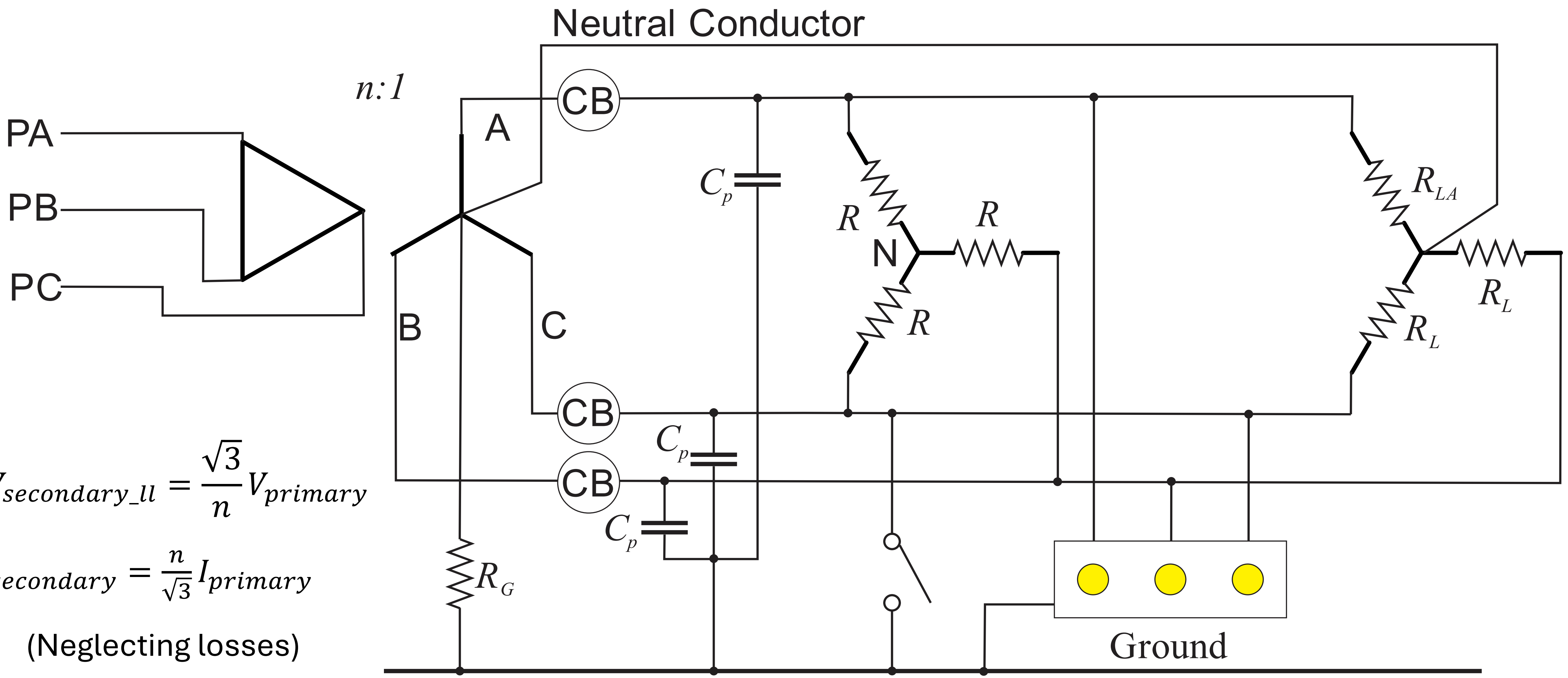
Ground Fault Currents



Delta-Wye 4-wire Summary (solid ground)

- Effective turns ratio: $\frac{n}{\sqrt{3}}$ with 30° phase shift.
- Unbalanced delta loads impact magnitude of phase currents and their phase angles.
  - Phase currents DO NOT add to zero: current in neutral conductor
- Cable connected to wye winding can have a common-mode current
  - Equal to the current in the neutral conductor
- Common-mode current in wye-winding transformed to a circulating current in the delta winding
  - No common-mode current in the cable connected to the delta winding
- Ground Fault on solidly grounded system (wye connection):
  - Large Fault Current on grounded phase – circuit breaker trips
  - CM voltage and currents

Delta-Wye Transformers
4-wire high resistance grounded (HRG) secondary



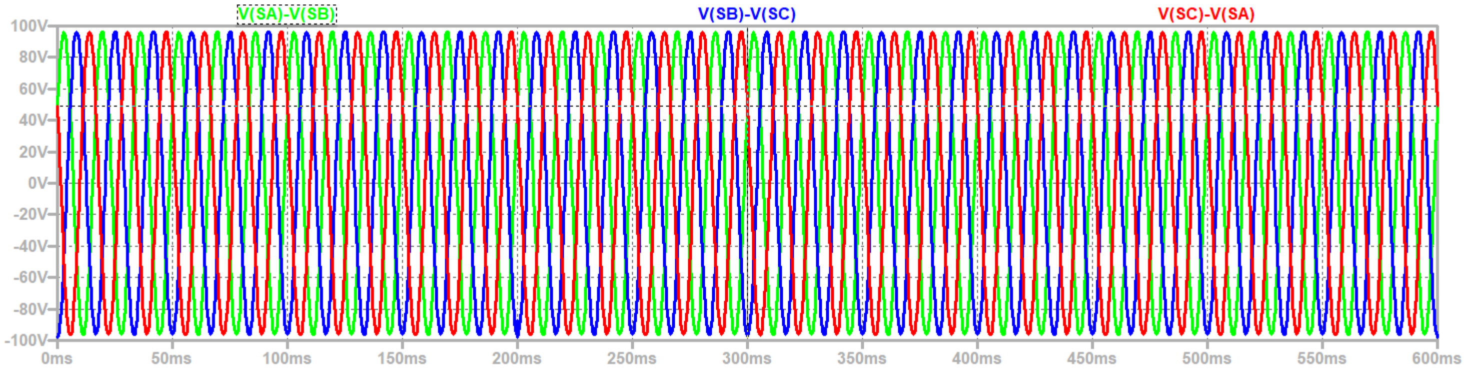
$$V_{secondary\_ll} = \frac{\sqrt{3}}{n} V_{primary}$$

$$I_{secondary} = \frac{n}{\sqrt{3}} I_{primary}$$

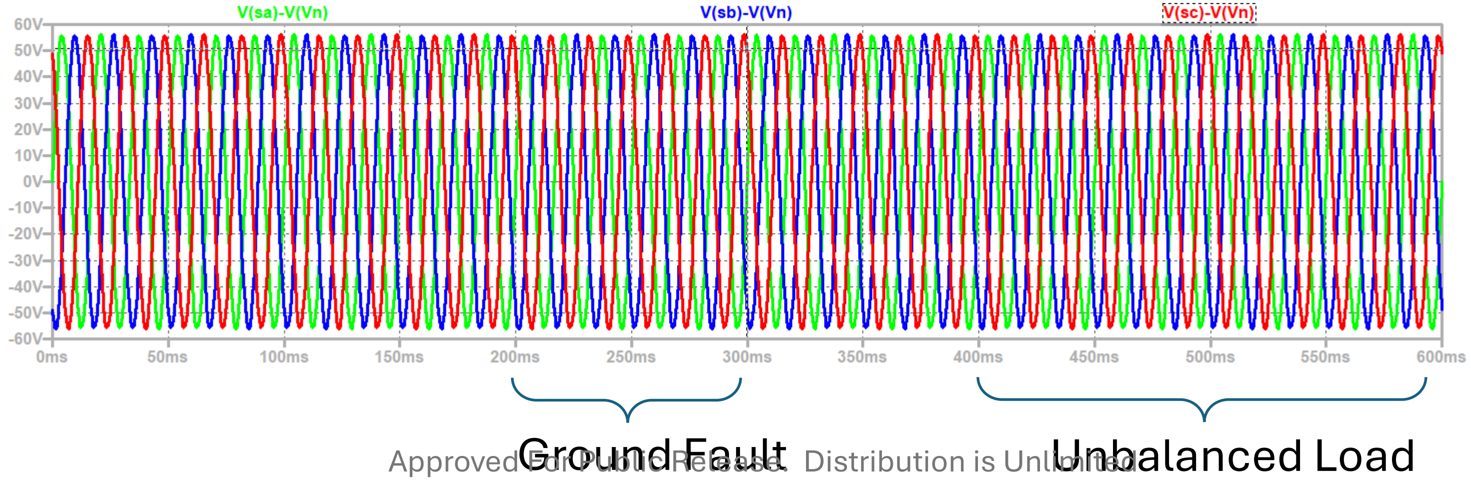
(Neglecting losses)

Steady State Voltages – Ground Fault / Unbalanced Load

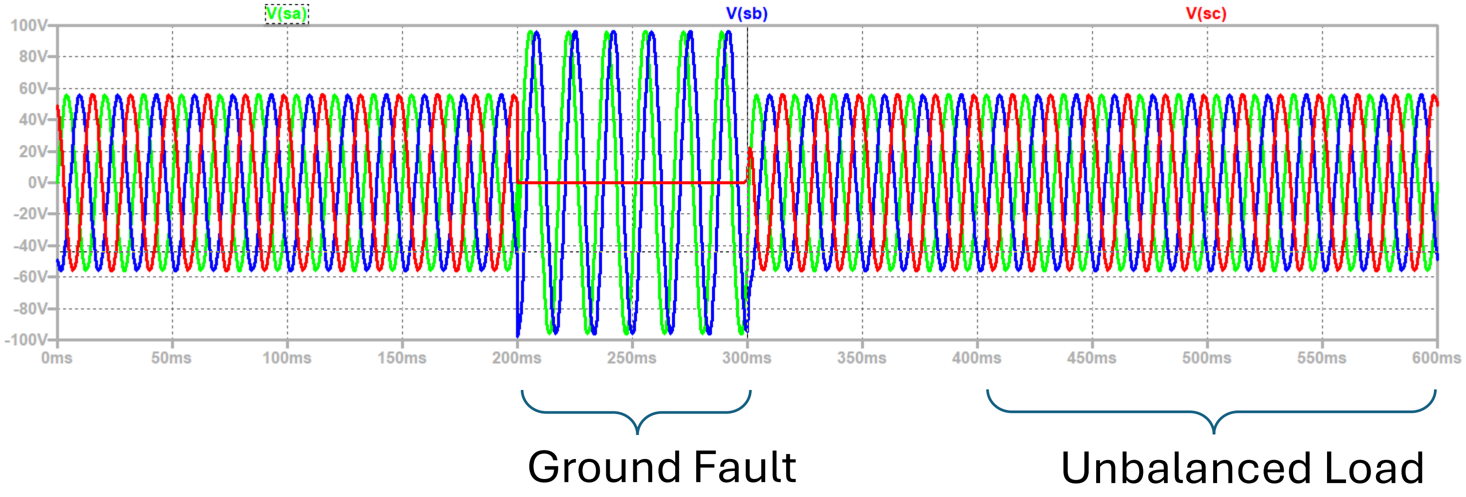
$$V_{secondary\_ll} = \frac{\sqrt{3}}{n} V_{primary} = \frac{\sqrt{3}}{5.2} 208 = 69.3\ volts\ rms\ \ (98\ V\ peak)$$



Secondary Line to Neutral Voltage: Peak = 56.6 V volts

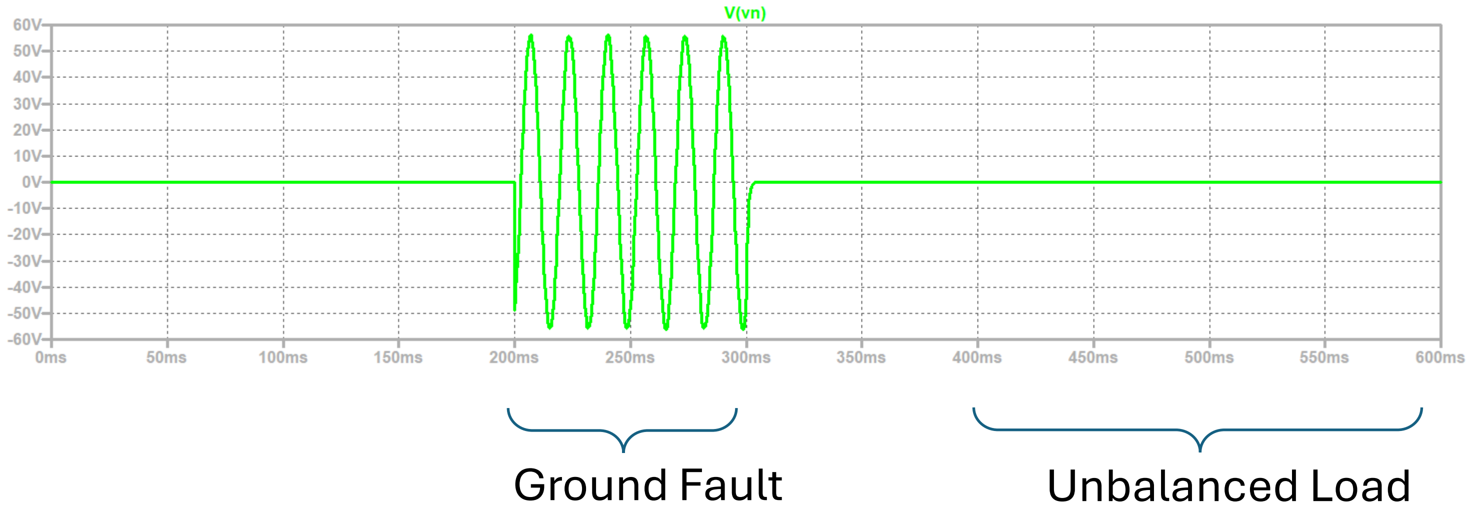


Steady State line to ground Voltages – Ground Fault / Unbalanced Load

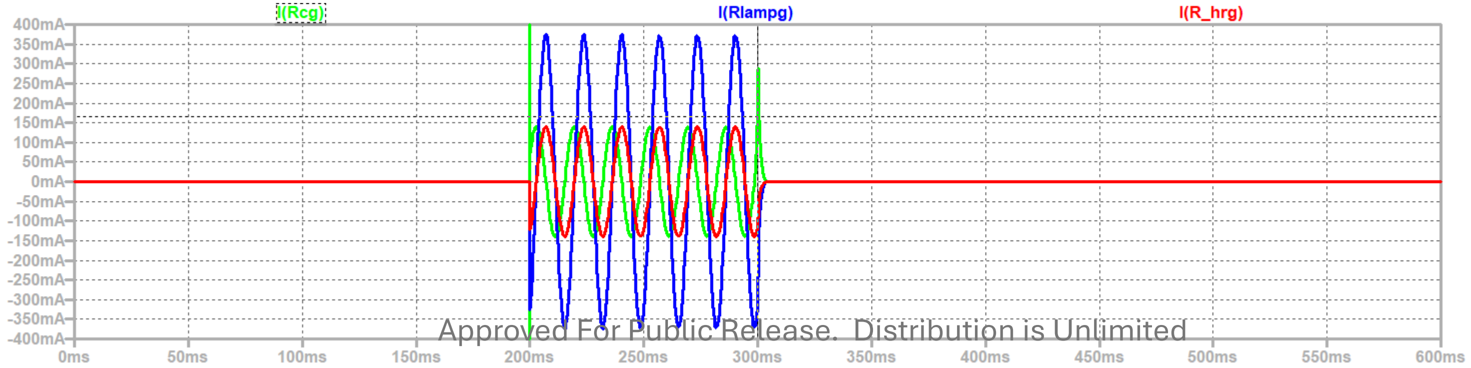


Neutral to ground voltage (CM Voltage) and CM Current through Parasitic Capacitances, lamp resistances, and grounding resistor

Neutral to Ground Voltage

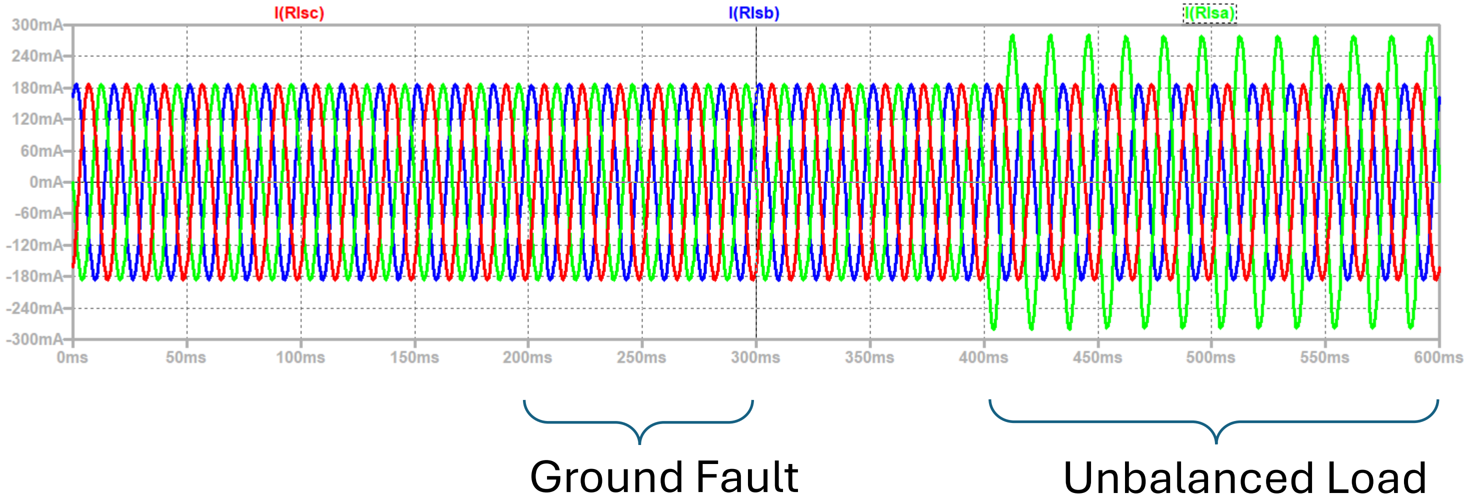


CM Current through Parasitic Capacitances, Lamp Resistors, and HRG resistor

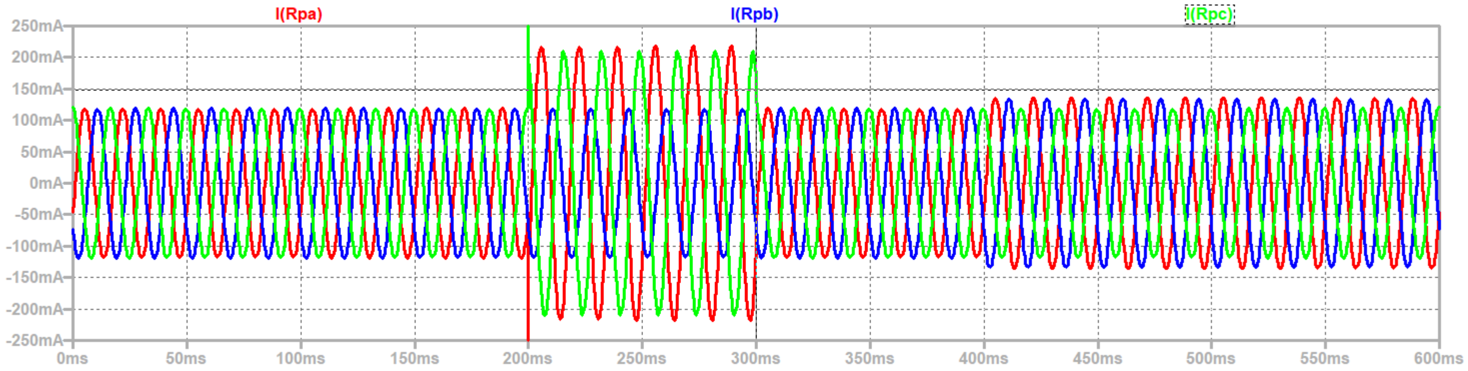


Line currents into the wye load and into primary not balanced with unbalanced load

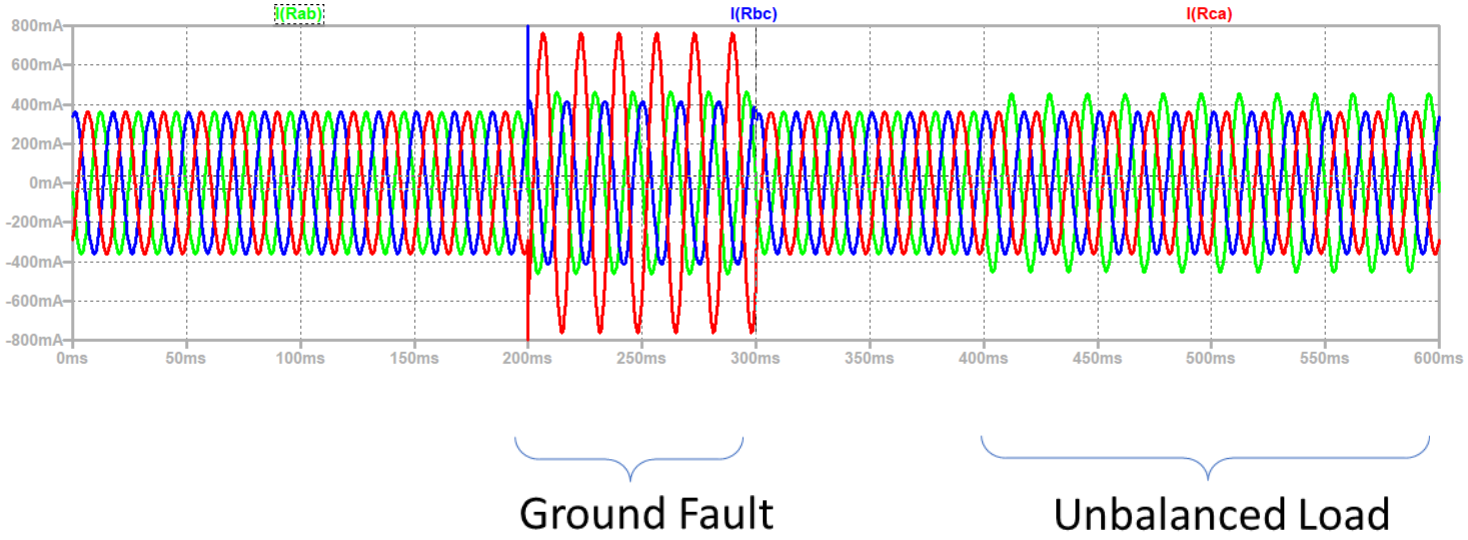
Current into Wye load



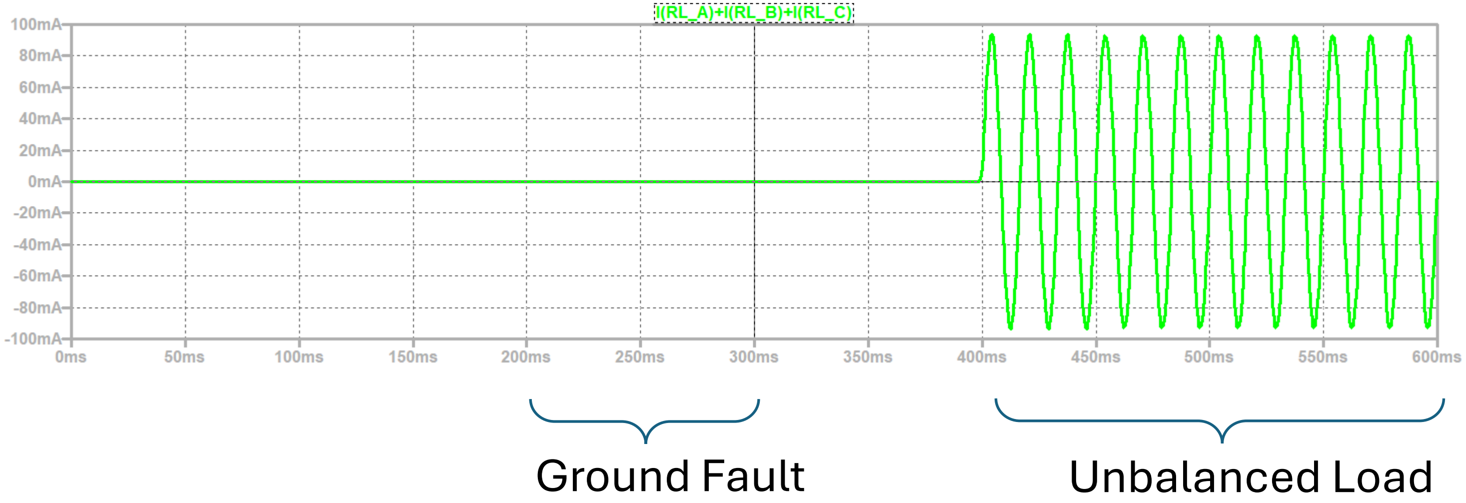
Current into Primary



Line Currents out of Transformer



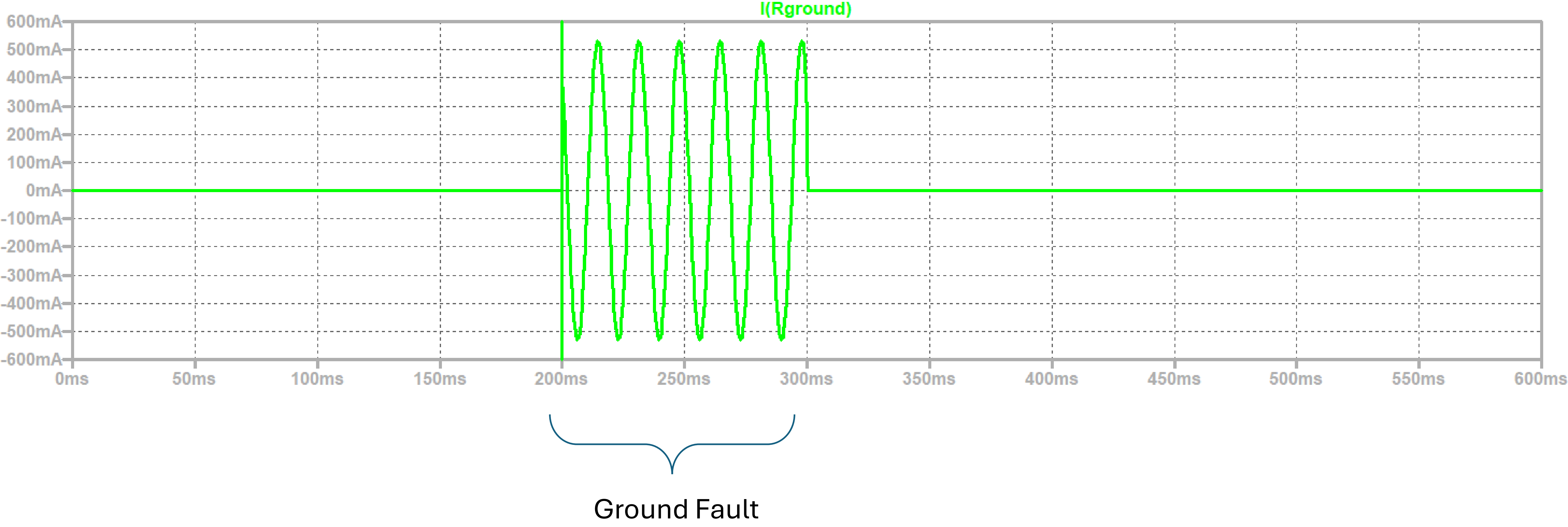
Unbalanced load results in current in the Neutral Conductor



For unbalanced loads

$$i_{neutral\_conductor} = \frac{\frac{98}{1.73}}{600} = 94.3 \text{ mA peak}$$

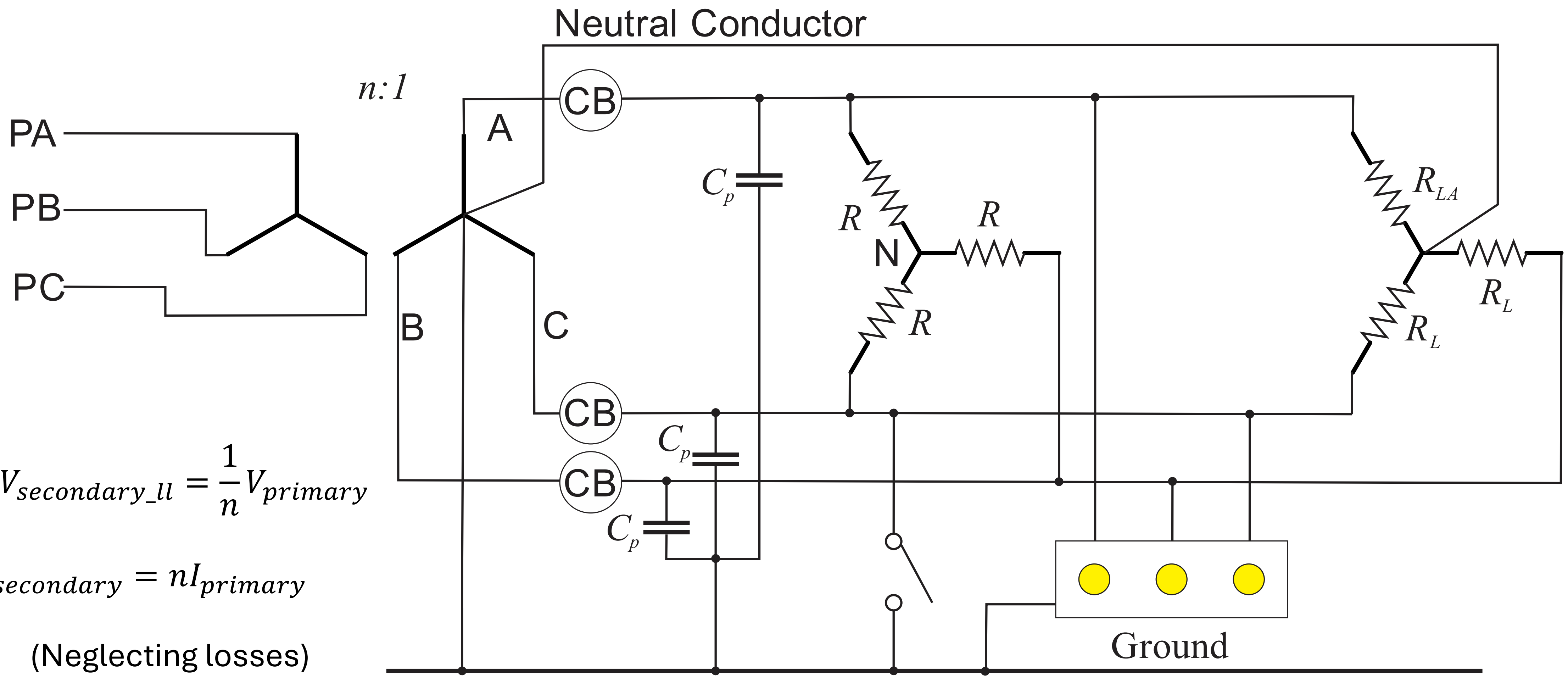
Ground Fault Currents



Delta-Wye 4-wire Summary (HRG)

- Effective turns ratio: $\frac{n}{\sqrt{3}}$ with 30° phase shift.
- Unbalanced delta loads impact magnitude of phase currents and their phase angles.
  - Phase currents DO NOT add to zero: current in neutral conductor
- Phase conductors connected to wye winding can have a common-mode current
  - Equal to the current in the neutral conductor
- Common-mode current in wye-winding transformed to a circulating current in the delta winding
  - No common-mode current in the cable connected to the delta winding
- Ground Fault with HRG system (wye connection):
  - Low Fault Current on grounded phase – circuit breaker does not trip
  - CM voltage and currents

Wye-Wye Transformers
4-wire solidly grounded secondary



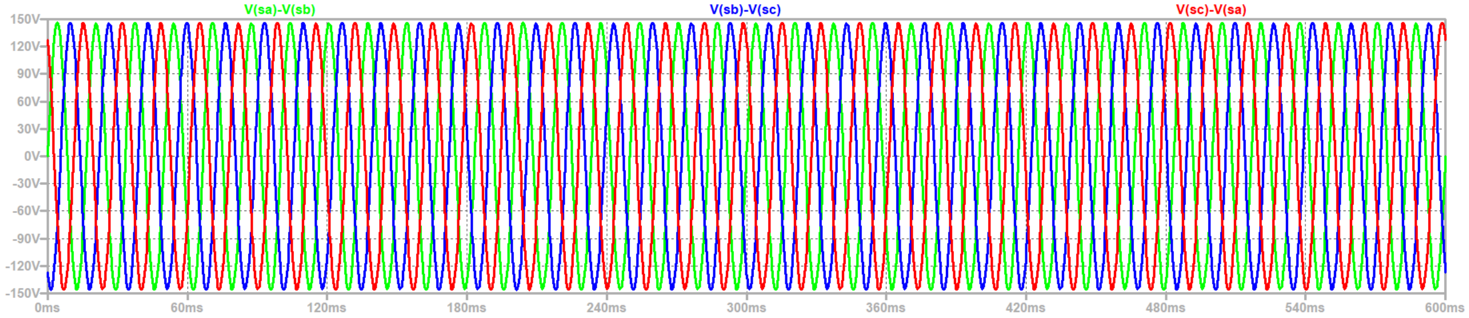
$$V_{secondary\_ll} = \frac{1}{n} V_{primary}$$

$$I_{secondary} = n I_{primary}$$

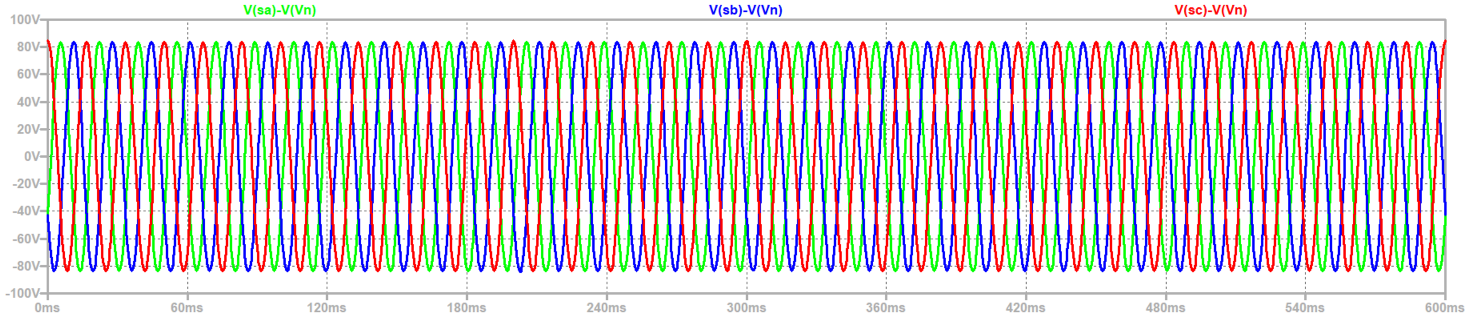
(Neglecting losses)

Steady State Voltages – Ground Fault / Unbalanced Load

$$V_{secondary\_ll} = \frac{1}{n} V_{primary} = \frac{1}{2} 208 = 104\ V\ rms\ \ (147\ V\ peak)$$



Secondary Line to neutral Voltage: 60 V rms (85 V peak)

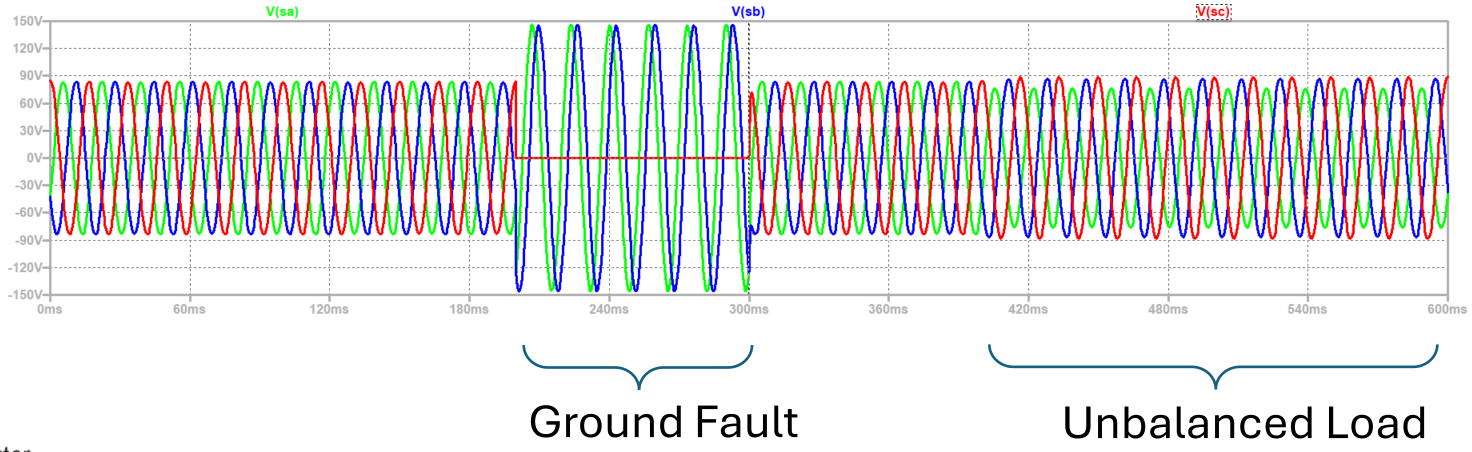


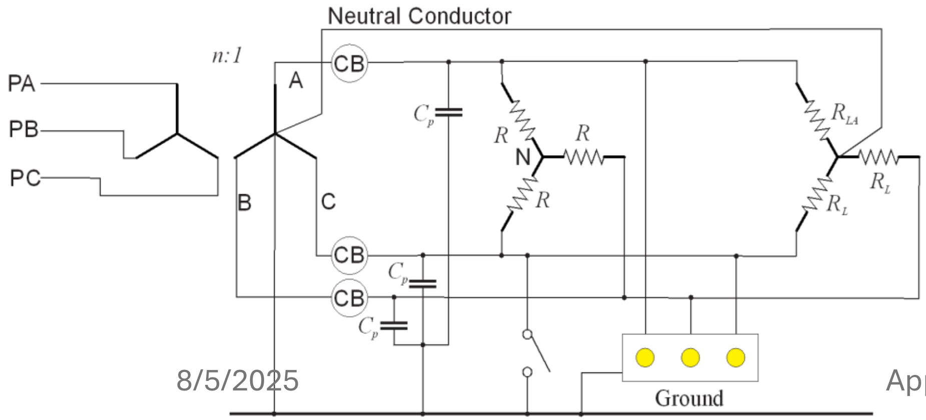
Ground Fault

Unbalanced Load

Steady State line to ground Voltages – Ground Fault / Unbalanced Load

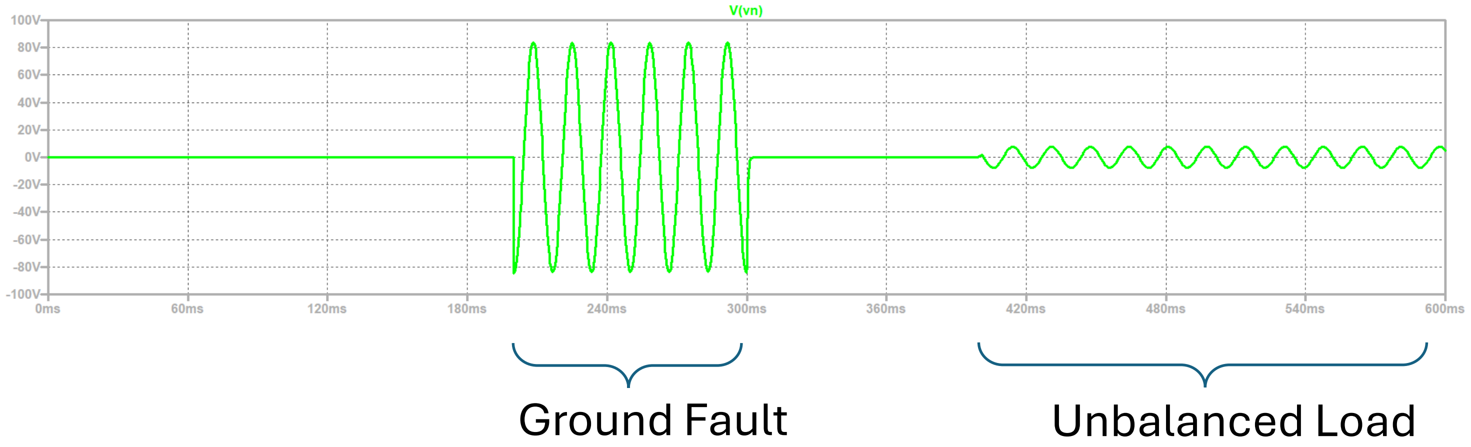
Loads on unfaulted phases experience 73% overvoltage during ground fault



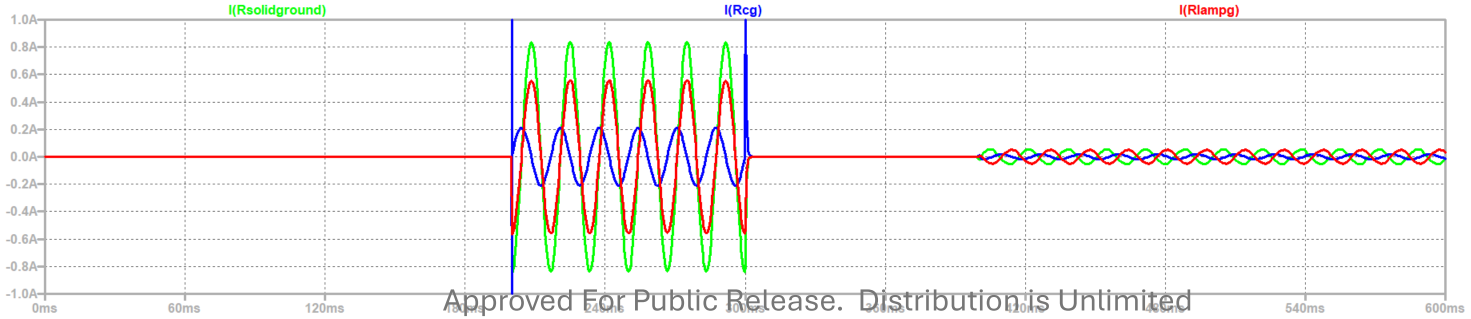


Neutral to ground voltage (CM Voltage) and CM Current through Parasitic Capacitances, lamp resistances, and grounding resistor

Neutral to Ground Voltage

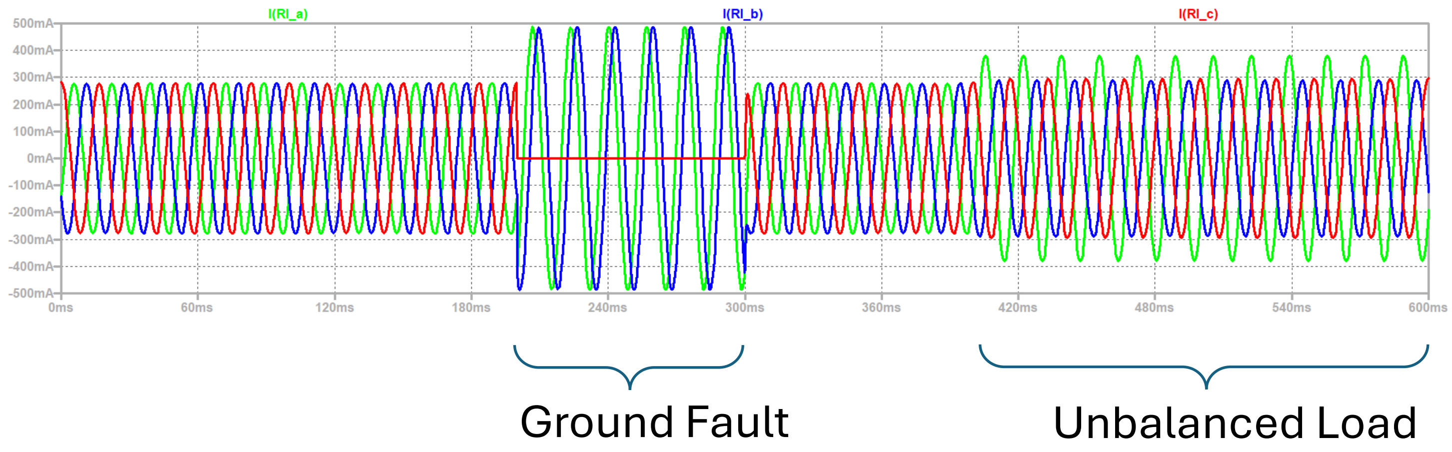


CM Current through Parasitic Capacitances, Lamp Resistors, and Ground Connection

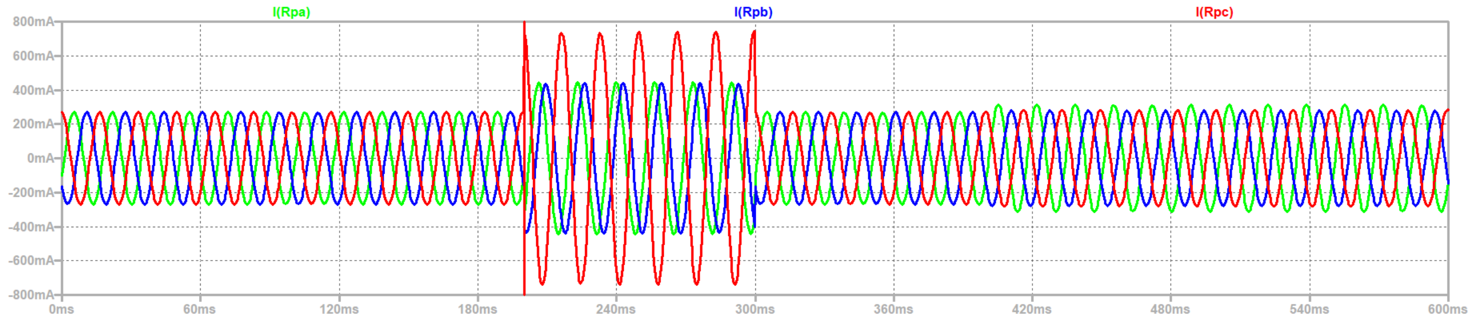


Line currents into the wye load and into primary not balanced with unbalanced load

Current into Wye load

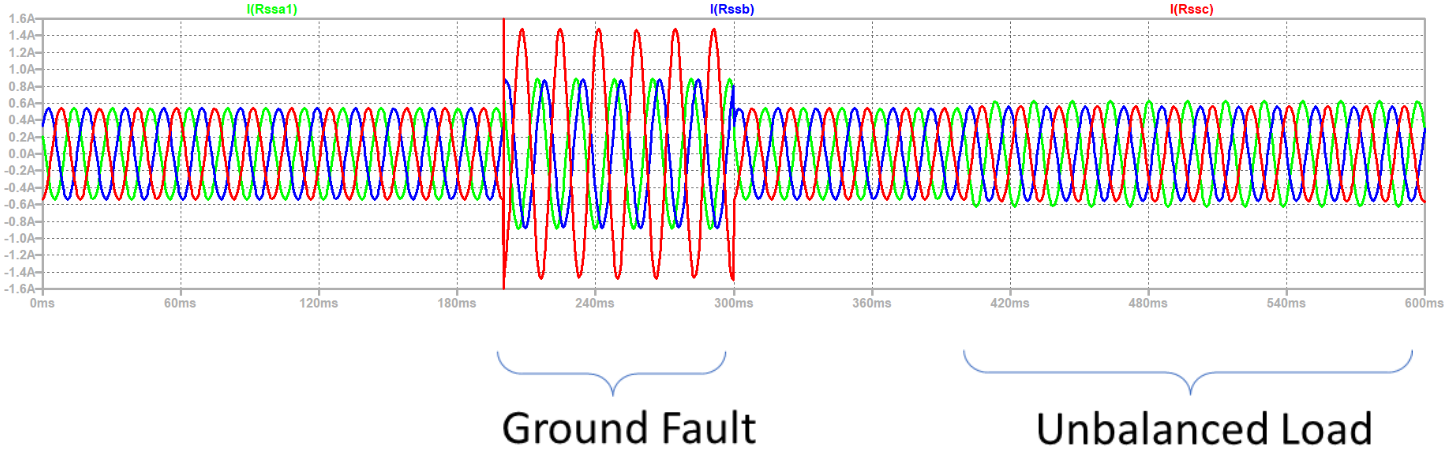


Current into Primary



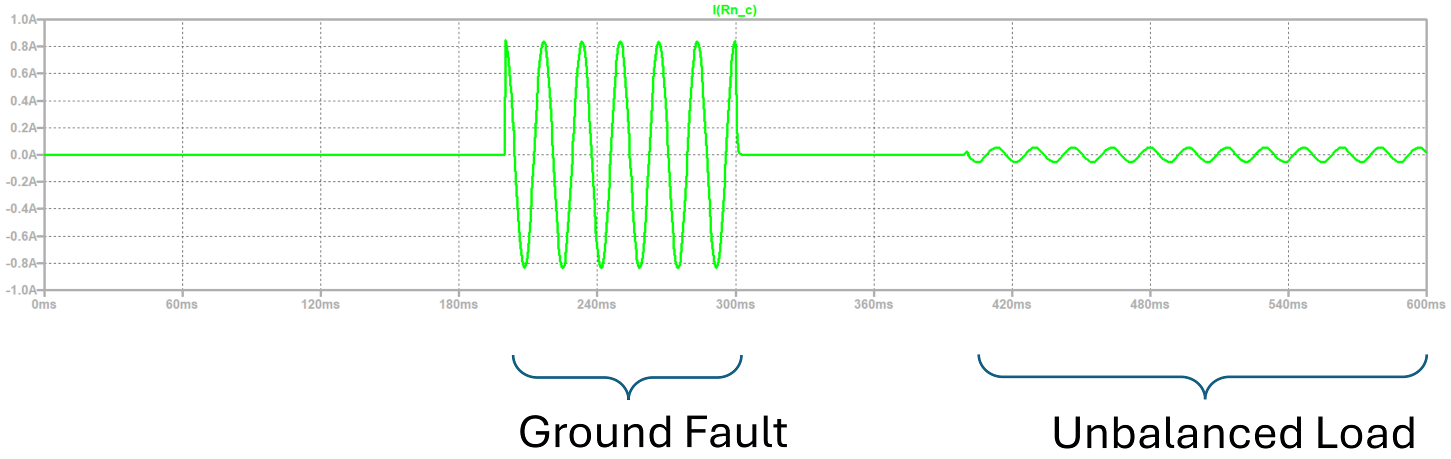
Primary currents not extremely large during ground fault – may not trip fault protection

Line Currents out of Transformer

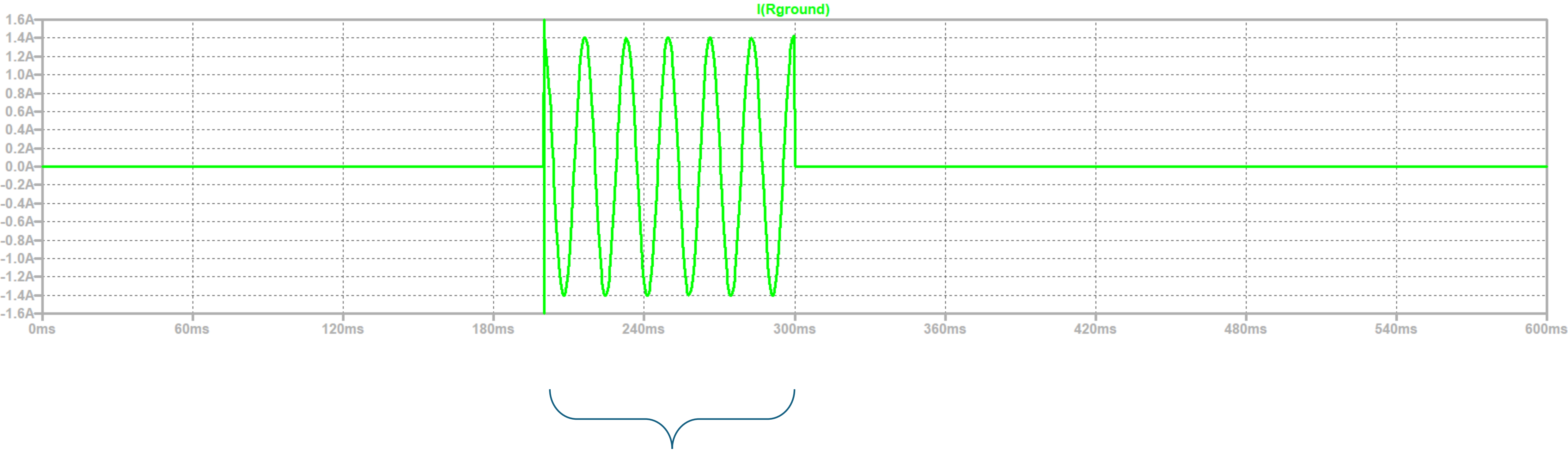


Secondary currents not extremely large during ground fault – may not trip fault protection

Ground faults and unbalanced load results in current in the Neutral Conductor



Ground Fault Currents



Ground fault current may not be Sufficient to trip breakers

Wye-Wye 4-wire Summary (solidly grounded)

- Effective turns ratio: n.
- Phase conductors connected to wye winding secondary cannot have a common-mode current
  - Unterminated primary common connection point forces sum of currents to be zero.
  - No current may flow into secondary common connected point
- Unbalanced delta loads impact magnitude of phase currents and their phase angles.
  - Phase currents DO NOT add to zero: current in neutral conductor
  - Current in neutral conductor does not flow into wye secondary
  - Current in neutral conductor flows through parasitic capacitances (ground lamps)
- Neutral conductor can have current between components other than transformer secondary.
- Ground Fault with solidly grounded system
  - Relatively Low Fault Current on grounded phase – circuit breaker may not trip
  - CM voltage and currents, but no CM current through transformer.

References

- http://doerry.org/norbert/MarineElectricalPowerSystems/videos/index.htm
  - Provides videos, Powerpoint, and LTSpice models
- Doerry, Norbert, "Shipboard Three-Phase Power Transformer Analysis," Version 1.0, December 10, 2022
  - http://doerry.org/norbert/MarineElectricalPowerSystems/references/P_20221210a/20221210%20Shipboard%20Three%20Phase%20Transformers.pdf
- Doerry, Norbert, Mohammed M. Islam, John Prousalidis, *Design of Shipboard Power System Grounding / Earthing* , IEEE Press - Wiley, January 2025, ISBN-10 1119933080 ISBN-13 978-1119933083